From simple dynamics to chaos through nonmonotone delayed feedback

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Some seemingly simple nonlinear delay differential equations still pose massive problems to the understanding of their global dynamics, even after many decades of intensive research. In the talk an overview of some recent results will be presented for two celebrated model equations with unimodal feedback: the Nicholson blowflies equation arisen in population dynamics, and the Mackey-Glass equation which has been proposed to model blood cell production and haematological diseases, and well known for its chaotic behavior. In particular, we give conditions that ensure that all solutions eventually enter the domain where the feedback is monotone, thus chaotic behavior can be excluded. We give sharp (in certain sense the sharpest) bounds for the global attractor and construct heteroclinic orbits from the trivial equilibrium to a slowly oscillating periodic orbit around the positive equilibrium. We discuss the coexistence of rapidly oscillating periodic solutions, and provide many numerical examples for different scenarios.

Keywords: delay differential equation, nonlinear dynamics, chaotic behavior, global attractor

References

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