Potential theory on curves

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The purpose of this talk is to show some old and new results in potential theory. Recently, polynomial inequalities have been proved in various settings and various classes of sets. See, for example, Sarantopoulos1991, Totik2000, NagyTotik2005. Roughly speaking, first they prove the polynomial inequalities on nice sets (ellipses, real lemniscates or complex lemniscates), then using an approximation argument, transfer the results to a wider class of sets. During this transfer, by a simple containing relation, supremum norms are preserved. This transfer is not possible in more general setting, since this containing relation is too restrictive. This is why I investigate potential theory on curves.

By curves, I mean a finite system of Jordan curves. Further assumptions are sometimes needed, for example, each of these curves is simple, they do not intersect each other and they are smooth enough. In any case, this is a compact set K. A real lemniscate L is the inverse image of the $[-1, 1]$ interval via a polynomial r, that is, $L = r^{-1}[-1, 1]$. We use (logarithmic) potential theory and Green's functions. We measure the approximation in terms of $d(L, K) = \sup g_K(z) : z \in L$ where $g_K(z) = g_{\mathbf{C}_\infty \setminus K}(z, \infty)$ is the Green's function of the unbounded component of the complement of K. Using the smoothness of K, we estimate the asymptotic behaviour $d(L, K)$ with the degree of r.