Billiards and Random Walks

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Mathematical theory of billiards is a fascinating subject with an enormous progression in the last decades. With the recently developed technics, lots of the arguments used in stochastics (mainly in the theory of random walks) can be adapted to the case of such deterministic systems.

For instance, the so-called Lorentz process is a deterministic model of the motion of an electron in metals: a freely moving point in the two dimensional space collides with fix, periodically situated scatterers and bounces off according to the classical law of mechanics (the angle of incidence is equal to the angle of reflection). The movement could be modeled with Simple Symmetric Random Walk (SSRW) - in this case the normality of the diffusively scaled limit is not difficult to prove. The same limes (i.e. Brownian motion) is by now proven for the Lorentz process.

However, owing to the complexity of the methods, lots of interesting questions remain open. For example, consider a locally perturbed Lorentz process, that is finitely many scatterers are replaced by somehow different ones. In this case one could expect the same limit process to appear.

This question for SSRW was treated by D. Szász and A. Telcs in 1981, and for Lorentz process with finite horizon by D. Dolgopyat, D. Szász an T. Varjú in 2009. Here, the assumption on finite horizon (i.e. when the free flight vector of the electron in bounded) is essential. In the case of infinite horizon, other interesting phenomena appear. For example, the scaling is super-diffusive (that is $\sqrt{n \log n}$ instead of $\sqrt{n}$). In this talk, we are going to focus on the treatment of the local perturbation in the case of infinite horizon on the random walk level, that is, we need to consider a distribution in the non normal domain of attraction of the Gaussian law. Among others, we need to estimate the probability of flying over a point in $\mathbb{Z}^2$, as the perturbation can modify the trajectory in such a case, as well.