Separated matchings in colored convex sets *

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There is a two-colored point set of $2n$ points, n points red and n points blue, in the plane in convex position. Without loss of generality we may assume that the points are on the circle C. Erdős asked to estimate the number of vertices on the longest non-crossing, alternating path. Kynčl, Pach and Tóth proved the upper bound $\frac{4}{3}n + c'\sqrt{n}$ for certain coloring and the lower bound $n + c'\sqrt{\frac{n}{logn}}$ for arbitrary colored point set where c and c' are positive constants. The upper bound is conjectured to be tight.

In this talk we concentrate on separated matchings, a notion closely related to the original problem. A *separated matching* is a matching where no two edges cross geometrically and all edges can be crossed by a line. It is believed that the size (number of matched points) of the maximum separated matching must be at least $\frac{4}{3}n$. We support this conjecture with a broad class of examples. Our examples also exhibit the currently known best upper bound for the Erdős problem.

Our construction contains two types of arcs that we repeat along the circle in arbitrary order an equal number of times. We show that this class of configurations allows at most $\frac{4}{3}n+O(\sqrt{n})$ vertices in the maximum separated matching. In the previous papers only one example was given with a coloring of high discrepancy. Our example suggests that the discrepancy doesn't play an important role. We investigate the case of coloring with small discrepancy. If the discrepancy is two or three, we show that the number of vertices in the maximum separated matching is at least $\frac{4}{3}n$.

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