Extended abstract

Single and double sine series with general monotonic coefficients

Kórus Péter
Bolyai Institute, University of Szeged
korpet@math.u-szeged.hu

One of the basic theorems in the theory of uniform convergence of sine series is due to Chaundry and Jolliffe (1916). They proved that if $\{a_k,\, k = 1, 2, \ldots\}$ is a nonnegative sequence converging monotonically to zero, then the series $\sum a_k \sin kx$ converges uniformly in x if and only if $ka_k \to 0$ $(k \to \infty)$. Recently a number of papers were published to extend this theorem by enlarging the class of monotonic coefficients, while keeping the condition $ka_k \to 0$ still necessary and sufficient. Some of the class conditions are the following:

$$\mathrm{RBV}: \sum_{k=n}^{\infty} |\Delta a_k| \le C a_n; \qquad \mathrm{GBV}: \sum_{k=n}^{2n} |\Delta a_k| \le C \max_{n \le k \le n+N_0} a_k;$$

$$\mathrm{NBV}: \sum_{k=n}^{2n} |\Delta a_k| \le C(|a_n| + |a_{2n}|); \qquad \mathrm{MVBV}: \sum_{k=n}^{2n} |\Delta a_k| \le \frac{C}{n} \sum_{k=[\lambda^{-1}n]}^{[\lambda n]} |a_k|.$$

In the theorems involving classes NBVS and MVBVS, the sufficiency part is proved for sine series with complex coefficients as well, while the necessity part stands for sine series with nonnegative coefficients. The following two classes of appropriate sequences of coefficients were introduced by me quite recently:

$$\mathrm{SBVS}: \sum_{k=n}^{2n-1} |\Delta a_k| \le \frac{C}{n} \sup_{m \ge [n/\lambda]} \sum_{k=m}^{2m} |a_k|; \quad \mathrm{SBVS}_2: \sum_{k=n}^{2n-1} |\Delta a_k| \le \frac{C}{n} \sup_{m \ge b(n)} \sum_{k=m}^{2m} |a_k|$$

where $\{b(k) : k = 1, 2, \ldots\} \subset [0, \infty)$ tends monotonically to infinity. In the above enumeration, each class contains the previous classes as a subclass.

In the second part of my talk I present my new results on the uniform convergence of double sine series of the form $\sum_{j=1}^{\infty} \sum_{k=1}^{\infty} a_{jk} \sin jx \sin ky$. In this case, the first known result was proved by Žak and Šneider (1966) for double sine series with monotonic double sequences of nonnegative numbers, which are defined by the conditions $a_{jk} \ge 0$, $\Delta_{10} a_{jk} \ge 0$, $\Delta_{01} a_{jk} \ge 0$, $\Delta_{11} a_{jk} \ge 0$. It was proved by them that such a double sine series is uniformly regularly convergent if and only if $jka_{jk} \to 0$ $(j + k \to \infty)$. To extend this result, larger classes than the monotonic ones – similar to the one variable case – have been defined in 2009, named MVBVDS and NBVDS. This time I shall introduce the notions SBVDS and SBVDS_2 and characterize the uniform convergence of the above double sine series with coefficients from these classes.