Capacity Region of the Discrete Asynchronous Multiple Acces Channel

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Briefly multiple access channel means that many senders send messages to one receiver simultaneously. For the sake of simplicity let us assume that there are two senders. The capacity region consists of the rate pairs (R_1, R_2), such that the first and second sender can send information simultaneously with rate R_1, R_2, respectively, with arbitrary small average probability. If the users are synchronous then the capacity region is the convex closure of the union of some pentagons. The method of time sharing makes possible to achieve the convex closure.

The asynchronous multiple acces channel (AMAC) arises when the senders do not synchronize the starting times of their codewords, rather there is an unkown delay between the senders. Cover [2] showed that if a bound b_n is present for the delay; depending on the codeword length n such that $\frac{b_n}{n} \to 0$ then the convex closure is still achievable by a generalised time sharing method. However, if the delay is not bounded then time sharing seems impossible, and it is widely accepted that in this case the union of pentagons is the capacity region. This view is supported by the papers [3, 4, 5]. However the assertion appears not have been proved completely.

Since the AMAC seems to be an important mathematical model our main purpose was to give a mathematically complete formulation of model and a full derivation of the capacity region.

It is shown by the technique of Gray from [1] and the techniques of Grant, Rimoldi and Urbanke [6] that the union is achievable regardless of the distributions of the delay. Our proof differs from [6] in that the delay is not known to the decoder. This setting is the same as in [4] by Hui and Humblet, but their proof contains errors. Note that [3, 5] assumed that the delay is known to the decoder.

More importantly a complete derivation of the converse is provided. Our new approach shows how the capacity region depends on the distribution of the delay. The familiar regions are recovered in the two classical cases when the delay is bounded, and when it is uniformly distributed. An interesting new case — where the delay is uniformly distributed over the even numbers — is presented. In this case the capacity region obtained is the convex combinations with weights $\frac{1}{2} - \frac{1}{2}$ of the pentagons.

References

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