Diffusive limit for the myopic (or "true") self-avoiding random walk in $d \geq 3$

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Abstract

The *myopic (or 'true') self-avoiding walk* model (MSAW) was introduced in the physics literature by Amit, Parisi and Peliti in 1983. It is a random motion in $\mathbb{Z}^d$ pushed towards domains less visited in the past by a kind of negative gradient of the occupation time measure: let w be a fixed smooth "rate function" and $t \mapsto X(t) \in \mathbb{Z}^d$ a continuous time nearest neighbor jump process on the integer lattice $\mathbb{Z}^d$ whose law is given as follows:

$$\mathbf{P}\big(X(t+\mathrm{d}t) = y \;\big|\; \text{past}, X(t) = x\big) = \begin{cases} w(\ell(t,x) - \ell(t,y))\,\mathrm{d}t + o(\mathrm{d}t) & \text{if } |x-y| = 1 \\ o(\mathrm{d}t) & \text{otherwise} \end{cases}$$

where

$$\ell(t,z) := \ell(0,z) + |\,\{0 \leq s \leq t : X(s) = z\}\,|\,, \qquad z \in \mathbb{Z}^d$$

is the occupation time measure of the walk $X(t)$ with some initial values $\ell(0,z)$, $z \in \mathbb{Z}^d$.

This is a continuous time version of the original *'true' self-avoiding random walk*. We investigate the asymptotic behaviour of MSAW in the non-recurrent dimensions. For a wide class of self-interaction functions, we identify a natural stationary (in time) and ergodic distribution of the environment (the local time profile) as seen from the moving particle and we establish diffusive lower and upper bounds for the displacement of the random walk. For a particular, more restricted class of interactions, we prove full CLT for the finite dimensional distributions of the displacement. This result settles part of the conjectures (based on non-rigorous renormalization group arguments) posed by Amit, Parisi and Peliti.

Key words and phrases: self-repelling random motion, local time, central limit theorem