On the commutant of asymptotically non-vanishing contractions

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One of the main methods of examining non-normal operators, acting on Hilbert spaces, is the theory of contractions. This area of operator theory was developed by Béla Szőkefalvi-Nagy and Ciprian Foias from the dilatation theorem of Béla Szőkefalvi-Nagy.

Szőkefalvi-Nagy and Foias classified the contractions according to their asymptotical behaviours. They got strong structural results in the case when the contraction and its adjoint are simultaneously asymptotically non-vanishing. However, basic questions are still open (e. g. the hyperinvariant and the invariant subspace problem), when we know only that the contraction is asymptotically non-vanishing. In this case one can associate a unitary asymptote to the contraction on a natural way. The connection with this unitary operator is manifested in an algebra homomorphism between the commutants of these operators.

Our purpose was to examine the injectivity of this commutant mapping. One of the results states that the commutant mapping can be injective even in the case when the contraction has a non-trivial stable subspace. Various characterizations of injectivity are provided.