STABILITY PROPERTIES OF A POPULATION DYNAMICAL MODEL FOR TWO FISH SPECIES IN LAKE TANGANYIKA

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This is a joint work with Professors László Hatvani and László L. Stachó. We investigate a population dynamical model initiated by László Stachó. This model describes the change of the amount of two fish species – a carnivore and a herbivore – living in Lake Tanganyika. The model consists of two parts: the development of the population during a year is described by a system of differential equations while the reproduction at the end of each year is described by a discrete dynamical system. The system of differential equations – after a series of simplifications – has the form:

$$\begin{aligned} \dot{L} &= C - LG \\ \dot{G} &= (L - \lambda(t))G, \end{aligned} \tag{1}$$

where $\lambda : [0, \infty) \to (0, \infty)$ is a given continuous function and $\lim_{t\to\infty} \lambda(t) = \lambda_1 > 0$ exists. This equation does not have an equilibrium, but its limit equation

$$\begin{aligned} \dot{L} &= C - LG \\ \dot{G} &= (L - \lambda_1)G, \end{aligned} \tag{2}$$

has the equilibrium $\left(\lambda_1, \frac{C}{\lambda_1}\right)$. We showed that $\left(\lambda_1, \frac{C}{\lambda_1}\right)$ is a globally eventually uniform-asymptotically stable point of the non-autonomous system (1). In the proof we use linearization, the method of limit equations and Lyapunov's direct method.

We also show some results about the behaviour of the whole system with the discrete part of the reproduction. We have also studied the slightly modified system where the growth of the vegetation left on its own would be exponential:

$$\begin{aligned} \dot{L} &= (C - G)L \\ \dot{G} &= (L - \lambda(t))G. \end{aligned} \tag{3}$$