Local degree distributions in scale free random graph models

Ágnes Backhausz, Eötvös Loránd University, Budapest
Supervisor: Tamás Móri

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Since the end of the nineties several complex real world networks and their random graph models were investigated. Many of them possess the scale free property: the tail of the degree distribution decreases polynomially fast, that is, if c_d denotes the proportion of vertices of degree d, then for large values of d $c_d \approx C \cdot d^{-\gamma}$ holds [1]. γ is called the characteristic exponent.

If the whole network is completely known, the empirical estimator of the characteristic exponent may have nice properties. However, real world networks usually are too large and complex, hence our knowledge of the graph is partial. For several models of evolving random graphs the degree distribution and the characteristic exponent change when attention is restricted to a set of selected vertices that are close to the initial configuration ([2, 3]).

Starting from these phenomena, the degree distribution constrained on a set of selected vertices is investigated, assuming that the graph model possesses the scale free property with characteristic exponent $\gamma > 1$, and the number of the selected vertices grows regularly with exponent $0 < \alpha \leq 1$. Sufficient conditions for the almost sure existence of asymptotic degree distribution are given. Loosely speaking, these conditions ensure that the degree of the new vertex is not too large and that the neighbors of the new vertex determine whether it belongs to the set of selected vertices or not. Moreover, it is shown that if these conditions hold, then the characteristic exponent of the constrained degree distribution is equal to $\alpha(\gamma - 1) + 1$. The proofs are based on the methods of martingale theory [4].

We present several graph models that satisfy the sufficient conditions, e. g. generalizations of the Barabási tree, random multitrees. All of these models possess the preferential attachment property, and in each model, the characteristic exponent of the constrained degree distribution is less then the original one. One reason for that is the following: the selected vertices are closer to the initial configuration in some sense. There are more "old" vertices among them and their degree is larger than that of the "typical" ones.

We also present a few random graph models showing the neccessity of some of the conditions.

References

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