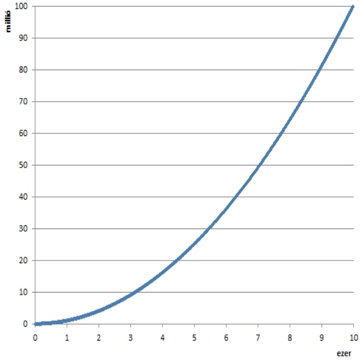
Goodstein-sorozatok

Goodstein-sorozat: $a_0 = 4$

$$
\begin{aligned}
a_0 &= 2^2 & &= 4 \\
a_1 &= 3^3 - 1 & &= 26 & = \\
&= 2 \cdot 3^2 + 2 \cdot 3 + 2 \\
a_2 &= 2 \cdot 4^2 + 2 \cdot 4 + 2 - 1 & &= 41 & = \\
&= 2 \cdot 4^2 + 2 \cdot 4 + 1 \\
a_3 &= 2 \cdot 5^2 + 2 \cdot 5 + 1 - 1 & &= 60 & = \\
&= 2 \cdot 5^2 + 2 \cdot 5 \\
a_4 &= 2 \cdot 6^2 + 2 \cdot 6 - 1 & &= 83 \\
&= 2 \cdot 6^2 + 6 + 5 \\
a_5 &= 2 \cdot 7^2 + 7 + 5 - 1 & &= 109 & = \\
&= 2 \cdot 7^2 + 7 + 4 \\
a_6 &= 2 \cdot 8^2 + 8 + 4 - 1 & &= 139 & = \\
&= 2 \cdot 8^2 + 8 + 3 \\
a_7 &= 2 \cdot 9^2 + 9 + 3 - 1 & &= 173 & = \\
&= 2 \cdot 9^2 + 9 + 2 \\
a_8 &= 2 \cdot 10^2 + 10 + 2 - 1 & &= 211 & = \\
&= 2 \cdot 10^2 + 10 + 1 \\
a_9 &= 2 \cdot 11^2 + 11 + 1 - 1 & &= 253 & = \\
&= 2 \cdot 11^2 + 11 \\
a_{10} &= 2 \cdot 12^2 + 12 - 1 & &= 299
\end{aligned}
$$

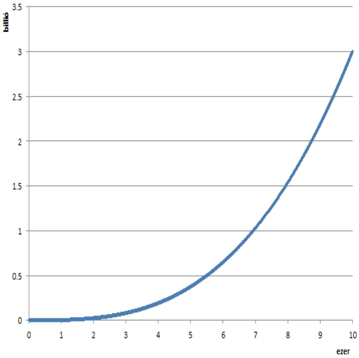
Goodstein-sorozat: $a_0 = 4$

$$
\begin{aligned}
a_0 &= 4\\
a_1 &= 26\\
a_2 &= 41\\
a_3 &= 60\\
a_4 &= 83\\
a_5 &= 109\\
a_6 &= 139\\
a_7 &= 173\\
a_8 &= 211\\
a_9 &= 253\\
a_{10} &= 299\\
a_{20} &= 969\\
a_{30} &= 1775\\
a_{40} &= 2735\\
a_{50} &= 3891\\
a_{60} &= 5241\\
a_{70} &= 6791\\
a_{80} &= 8541\\
a_{90} &= 10491\\
a_{100} &= 12635\\
a_{200} &= 45025\\
a_{300} &= 97325\\
a_{400} &= 169067\\
a_{500} &= 261807\\
a_{600} &= 374007\\
a_{700} &= 506207\\
a_{800} &= 658373\\
a_{900} &= 830473\\
a_{1000} &= 1022573
\end{aligned}
$$



Goodstein-sorozat: $a_0 = 5$

$$
\begin{aligned}
a_0 &= 5\\
a_1 &= 27\\
a_2 &= 255\\
a_3 &= 467\\
a_4 &= 775\\
a_5 &= 1197\\
a_6 &= 1751\\
a_7 &= 2454\\
a_8 &= 3325\\
a_9 &= 4382\\
a_{10} &= 5643\\
a_{20} &= 33427\\
a_{30} &= 101407\\
a_{40} &= 227577\\
a_{50} &= 429947\\
a_{60} &= 726517\\
a_{70} &= 1134703\\
a_{80} &= 1672763\\
a_{90} &= 2358823\\
a_{100} &= 3210883\\
a_{200} &= 24821409\\
a_{300} &= 82831863\\
a_{400} &= 195242263\\
a_{500} &= 380052663\\
a_{600} &= 655262973\\
a_{700} &= 1038873273\\
a_{800} &= 1548883573\\
a_{900} &= 2203293873\\
a_{1000} &= 3020104173
\end{aligned}
$$



Goodstein-sorozat: $a_0 = 15$

$$
\begin{aligned}
a_0 &= 15 \\
a_1 &= 111 \\
a_2 &= 1283 \\
a_3 &= 18752 \\
a_4 &= 326593 \\
a_5 &= 6588344 \\
a_6 &= 150994943 \\
a_7 &= 3524450280 \\
a_8 &= 100077777775 \\
a_9 &= 3138578427934 \\
a_{10} &= 106993479003783 \\
a_{20} &= 7511413302012830262744519876673 \\
a_{30} &= 4676805239458889338251791464692105662923811819 4367 \\
a_{40} &= 630549937690245603898442283089868722819001476388702864390918152238132 3 \\
a_{50} &= 8875180874254872942662451563958081888110634839390750059066358378372882176469951448803713727 \\
a_{60} &\approx 8,330 \cdot 10^{112} \\
a_{70} &\approx 3,848 \cdot 10^{135} \\
a_{80} &\approx 7,023 \cdot 10^{158} \\
a_{90} &\approx 4,288 \cdot 10^{182} \\
a_{100} &\approx 7,688 \cdot 10^{206}
\end{aligned}
$$

Goodstein-sorozat: $a_0 = 16$

$$
\begin{aligned}
a_0 &= 16\\
a_1 &= 7625597484986\\
a_2 &= 5097399859121435513940637 7\\
a_3 &= 53793641718868912174424175024032593379100060\\
a_4 &= 199164895158705329602585621906393984715992390421859346480247611458 11\\
a_5 &= 51037084851229406318399011110368297914350076856673038724504351530153456868965305178143220707297 09\\
a_6 &\approx 1,619 \cdot 10^{132}\\
a_7 &\approx 1,056 \cdot 10^{174}\\
a_8 &\approx 2,220 \cdot 10^{222}\\
a_9 &\approx 2,251 \cdot 10^{277}\\
a_{10} &\approx 1,590 \cdot 10^{339}
\end{aligned}
$$

Goodstein-sorozat: $a_0 = 2020$

$$a_0 = 2020 = 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^2 =$$

$$= 2^{2^3+2} + 2^{2^3+1} + 2^{2^3} + 2^{2^2+2+1} + 2^{2^2+2} + 2^{2^2+1} + 2^2 =$$

$$= 2^{2^{2+1}+2} + 2^{2^{2+1}+1} + 2^{2^{2+1}} + 2^{2^2+2+1} + 2^{2^2+2} + 2^{2^2+1} + 2^2$$

$$a_1 = 3^{3^{3+1}+3} + 3^{3^{3+1}+1} + 3^{3^{3+1}} + 3^{3^3+3+1} + 3^{3^3+3} + 3^{3^3+1} + 3^3 - 1 =$$

$$= 3^{3^{3+1}+3} + 3^{3^{3+1}+1} + 3^{3^{3+1}} + 3^{3^3+3+1} + 3^{3^3+3} + 3^{3^3+1} + 2 \cdot 3^2 + 2 \cdot 3 + 2$$

$$= \quad 13\,746\,221\,135\,534\,170\,868\,395\,739\,395\,634\,967\,510\,476$$

$$a_2 = 4^{4^{4+1}+4} + 4^{4^{4+1}+1} + 4^{4^{4+1}} + 4^{4^4+4+1} + 4^{4^4+4} + 4^{4^4+1} + 2 \cdot 4^2 + 2 \cdot 4 + 2 - 1 =$$

$$\approx \quad 8,4347 \cdot 10^{618}$$

$$a_3 = 5^{5^{5+1}+5} + 5^{5^{5+1}+1} + 5^{5^{5+1}} + 5^{5^5+5+1} + 5^{5^5+5} + 5^{5^5+1} + 2 \cdot 5^2 + 2 \cdot 5 + 1 =$$

$$\approx 7,9800 \cdot 10^{10924}$$

Igaz, de nem bizonyítható

Tétel (Goodstein, 1944)

Minden $a_0 \in \mathbb{N}$ *kezdőérték esetén létezik olyan* N*, amelyre* $a_N = 0$*.*

Példa

$a_0 = 4$ esetén

$$N = 3 \cdot 2^{3 \cdot 2^{27} + 27} - 3 = 3 \cdot 2^{402\,653\,211} - 3 \approx 6,895 \cdot 10^{121\,210\,694}$$

Tétel (Kirby és Paris, 1982)

Goodstein tétele nem bizonyítható az elsőrendű Peano-aritmetikában.