
1 6 9 11
5 15 7 14
12 10 3 13
4 8 2

Samuel Loyd (1841-1911)

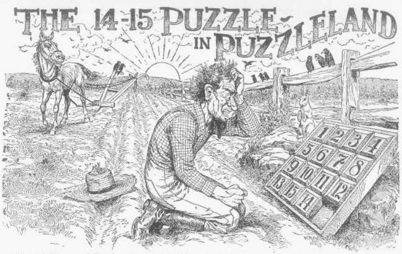
SAM LOYD'S CYCLOPEDIA

OF

5000 PUZZLES TRICKS AND CONUNDRUMS

WITH ANSWERS

THE 14-15 PUZZLE IN PUZZLELAND



The older inhabitants of Puzzleland will remember how in the early seventies I drove the entire world crazy over a little box of movable blocks which became known as the "14-15 Puzzle." The fifteen blocks were arranged in the square box in regular order, only with the 14 and 15 reversed, as shown in the above illustration. The puzzle consisted in moving the blocks about, one at a time, so as to bring them back to the present position in every respect except that the error in the 14 and 15 must be corrected.

A prize of $1,000, which was offered for the first correct solution to the problem, has never been claimed, although there are thousands of persons who say they performed the required feat.

People became infatuated with the puzzle and ludicrous tales are told of shopkeepers who neglected to open their stores; of a distinguished clergyman who stood under a street lamp all through a wintry night trying to recall the way he had performed the feat. The mysterious feature of the puzzle is that no one seems to be able to recall the sequence of moves whereby they feel sure they succeeded in solving the puzzle. Pilots are said to have wrecked their ships, engineers rush their trains past stations and business generally became demoralized. A famous Baltimore editor tells how he went for his noon lunch and was discovered by his frantic staff long past midnight pushing little pieces of pie around on a plate! Farmers are known to have deserted their plows and I have taken one of such instances as an illustration for the sketch.

Several new problems developed from the original puzzle which are worth giving:

Second Problem—Start again with the blocks as in Fig. 1 and move them so as to get the numbers in regular order, but with the vacant square at upper left-hand corner instead of lower right-hand corner; see Fig. 2.

Third Problem—Start with Fig. 1, turn the box a quarter way round and so move the blocks that they will rest as in Fig. 3.

Fourth Problem—This is to move the pieces about until they form a "magic square," so that the numbers will add up thirty in ten different directions.

Fig 2.

	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Fig 3.

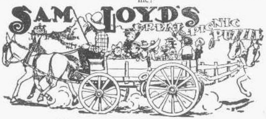
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The Picnic Puzzle.

When they started off on the great annual picnic every wagon in town was pressed into service. Half way to the grounds ten wagons broke down, so it was necessary for each of the remaining wagons to carry one more person.

When they started for home it was discovered that fifteen more wagons were out of commission, so on the return trip there were three persons more in each wagon than when they started out in the morning.

Now who can tell how many people attended the great annual picnic?



THE 14-15 PUZZLE IN PUZZLELAND

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AMERICAN

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DECEMBER, 1879.

Notes on the "15" Puzzle.

I.

By Wm. Woolsey Johnson, *Annapolis, Md.*

The puzzle described below has recently been exercising the ingenuity of many persons in Baltimore, Philadelphia and elsewhere. A ruled square of 16 compartments is numbered as in this diagram:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

the 16th square being left blank. Fifteen counters, numbered in like manner, are placed at random upon the squares so that one square is vacant. The counter occupying any adjacent square may now be moved into the vacant square—thus: If No. 7 is vacant, either of the counters occupying Nos. 3, 6, 8, 11 can be moved into it, but no diagonal move is allowed. The puzzle is to bring all the counters into their proper squares by successive moves.

It seems to be generally supposed, by those who have tried the puzzle, that this is always possible, whatever be the original random position of the counters, but this is an error, as the following demonstration will show:

When the blank or sixteenth square is the vacant one, the arrangement of the counters may be called a positive or negative one, according as the term of the 15-square determinant, which has for first and second subscripts the numbers on the squares and counters, is positive or negative. Let n

natural arrangement that which is obtained from the natural arrangement by reversing *all* the rows, leaving the left-hand square of the lower row blank. Using the notation just employed, the natural order may be reversed by $\frac{1}{2}cr - 1$ interchanges, when c is even; and by $\frac{1}{2}(c-1)r$ interchanges, when c is odd; *i. e.* the condition for the possibility of forming the reversed natural arrangement from any given arrangement will be the same or the opposite as that for the natural order, according as the number last obtained [$\frac{1}{2}cr - 1$, if c is even; and $\frac{1}{2}(c-1)r$, if c is odd] is even or odd. Thus if $r = 4$, $c = 4$; $\frac{1}{2}cr - 1 = 7$ and the reversed arrangement can always be formed when the natural arrangement cannot, and only then. If $r = 5$, $c = 5$; $\frac{1}{2}(c-1)r = 10$, and the reversed arrangement can be formed when the natural order can be, and only then. *I. e.* with the ordinary "15" puzzle, it is always possible to arrange the numbers in the natural order with the 1 in the right-hand upper square, when it is not possible to do it with the 1 in the left-hand upper square (as Mr. Johnson has remarked in the *postscript* to his note); but on a square board with five squares on a side, a solution is possible from each of the four corners, or not at all.

There are other arrangements beside the natural and reversed natural which may be regarded as solutions of the puzzle, viz: beginning at either corner of the board, the counters may be arranged in their natural order by rows or by columns, there being in all eight such solutions.

The "15" puzzle for the last few weeks has been prominently before the American public, and may safely be said to have engaged the attention of nine out of ten persons of both sexes and of all ages and conditions of the community. But this would not have weighed with the editors to induce them to insert articles upon such a subject in the American Journal of Mathematics, but for the fact that the principle of the game has its root in what all mathematicians of the present day are aware constitutes the most subtle and characteristic conception of modern algebra, viz: the law of dichotomy applicable to the separation of the terms of every complete system of permutations into two natural and indefeasible groups, a law of the inner world of thought, which may be said to prefigure the polar relation of left and right-handed screws, or of objects in space and their reflexions in a mirror. Accordingly the editors have thought that they would be doing no disservice to their science, but rather promoting its interests by exhibiting this *à priori* polar law under a concrete form, through the medium of a game which has taken so strong a hold upon the thought of the country that it may almost be said to have risen to the importance of a national institution. Whoever has made himself master of it may fairly be said to have taken his first lesson in the theory of determinants.

It may be mentioned as a parallel case that Sir William Rowan Hamilton invented, and Jacques & Co., the purveyors of toys and conjuring tricks, in London (from whom it may possibly still be procured), sold a game called the "Eikosion" game, for illustrating certain consequences of the method of quaternions.—EDS.

14	13	5	12
2	3	15	4
8		11	9
10	1	7	6

14	13	5	12
2	3	15	4
8		11	9
10	1	7	6

14	13	5	12	2	3	15	4	8		11	9	10	1	7	6

14	13	5	12
2	3	15	4
8		11	9
10	1	7	6

14	13	5	12
2		15	4
8	3	11	9
10	1	7	6

14 13 5 12 2 3 15 4 8 □ 11 9 10 1 7 6

14	13	5	12
2	3	15	4
8		11	9
10	1	7	6

14	13	5	12
2		15	4
8	3	11	9
10	1	7	6

14	13	5	12	2	3	15	4	8		11	9	10	1	7	6
14	13	5	12	2		15	4	8	3	11	9	10	1	7	6

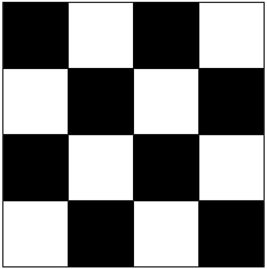
5 3 8 7 4 6 2 1

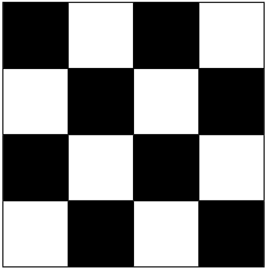
Rakjuk sorba cserék segítségével!

5   3   8   7   4   6   2   1

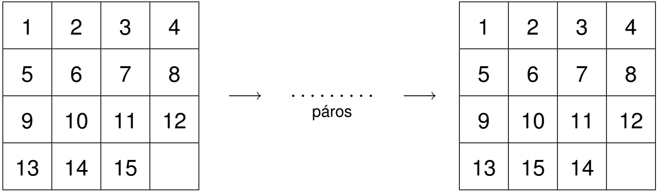
Rakjuk sorba cserék segítségével!

Aki az utolsó cserét végrehajtja, az a nyertes.

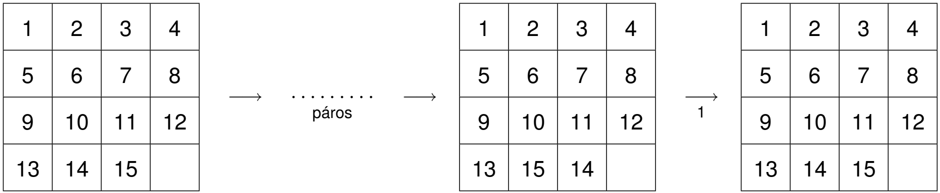




Ha az üres hely visszakerült a jobb alsó sarokba,
akkor páros számú csere történt.



1 2 3 4
5 6 7 8
9 10 11 12
13 14 15
páros
1 2 3 4
5 6 7 8
9 10 11 12
13 15 14

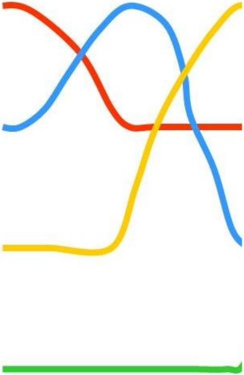

1 2 3 4
5 6 7 8
9 10 11 12
13 14 15
páros
1 2 3 4
5 6 7 8
9 10 11 12
13 15 14
1
1 2 3 4
5 6 7 8
9 10 11 12
13 14 15

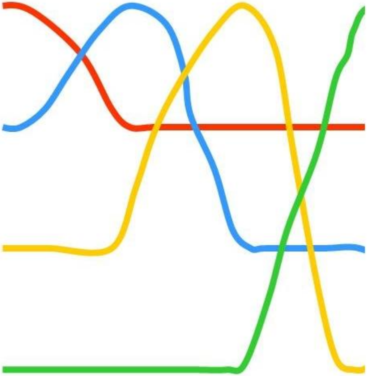
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

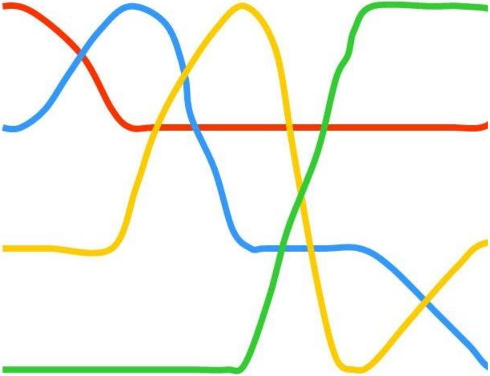
$\longrightarrow$ . . . . $\longrightarrow$

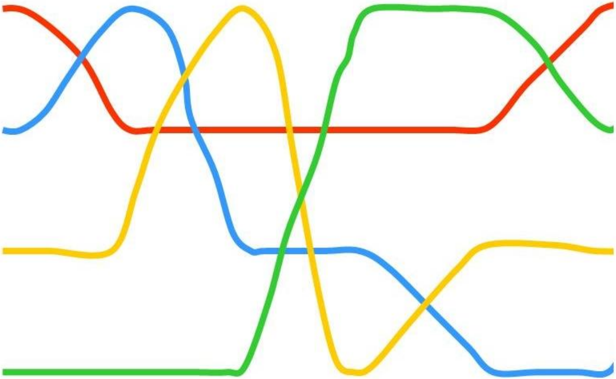
páratlan

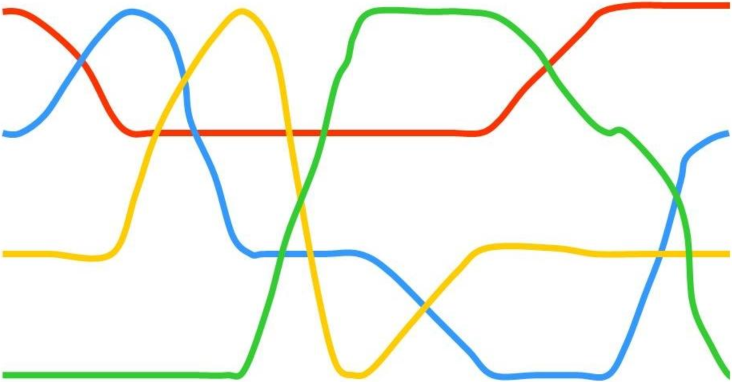
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

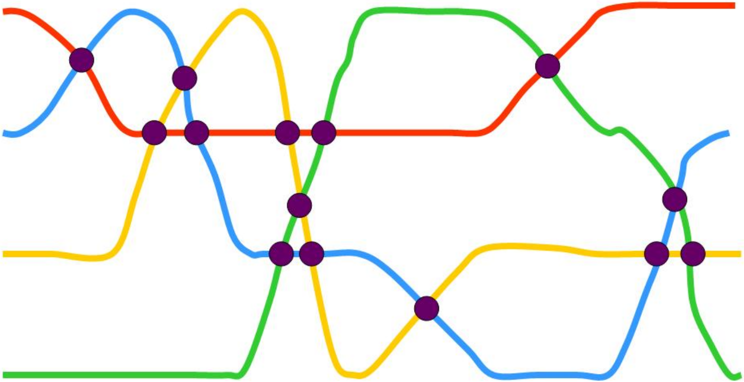


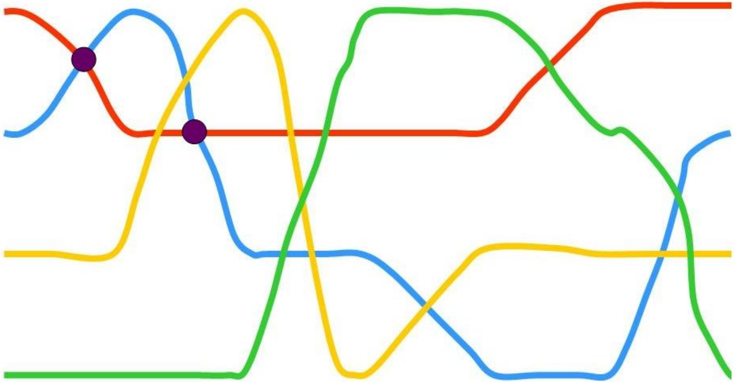




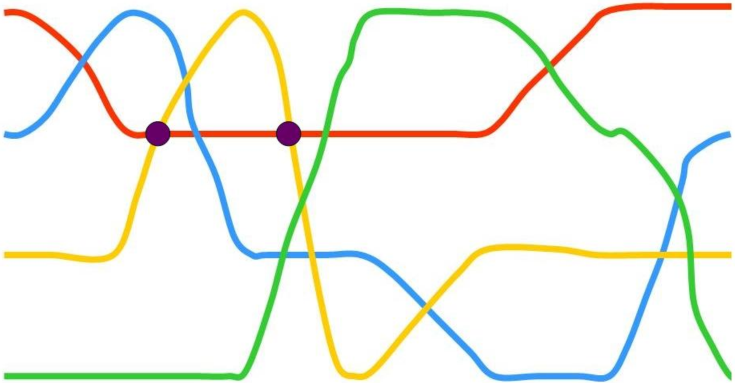




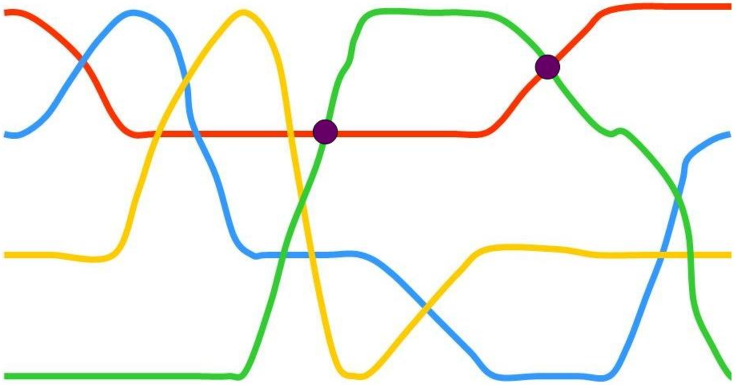




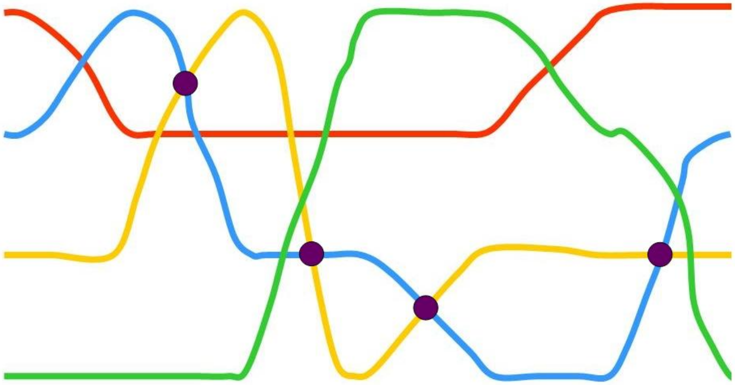
metszéspontok: 2



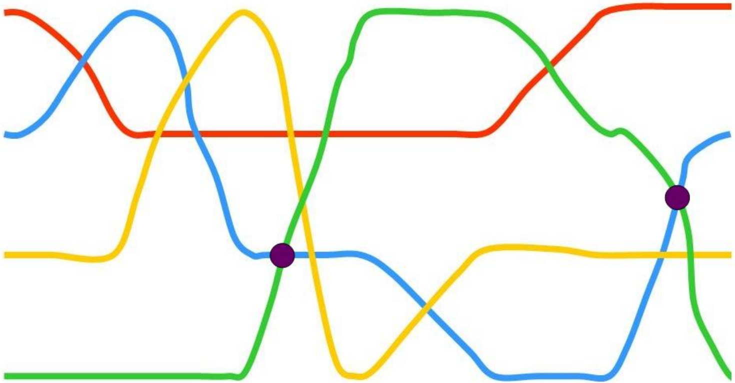
metszéspontok: $2 + 2$



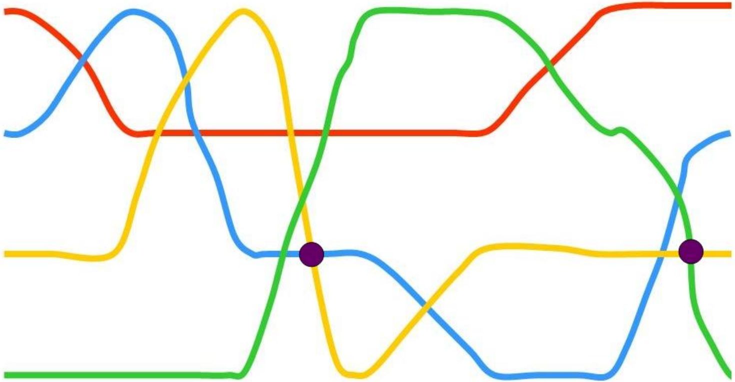
metszéspontok: $2 + 2 + 2$



metszéspontok: $2 + 2 + 2 + 4$

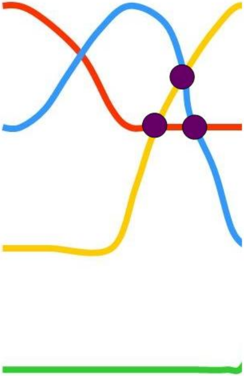


metszéspontok: $2 + 2 + 2 + 4 + 2$

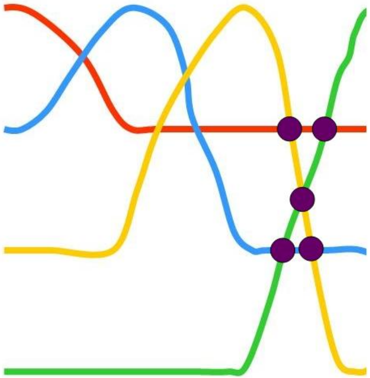


metszéspontok: $2 + 2 + 2 + 4 + 2 + 2 \equiv 0 \pmod 2$

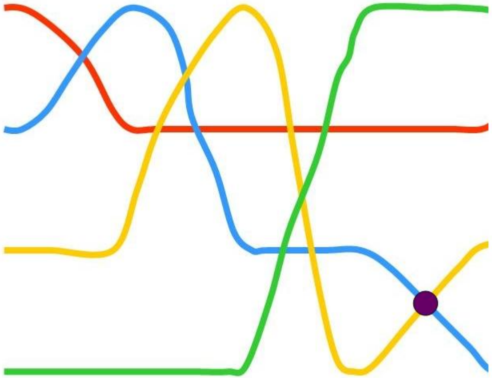
metszéspontok: 1



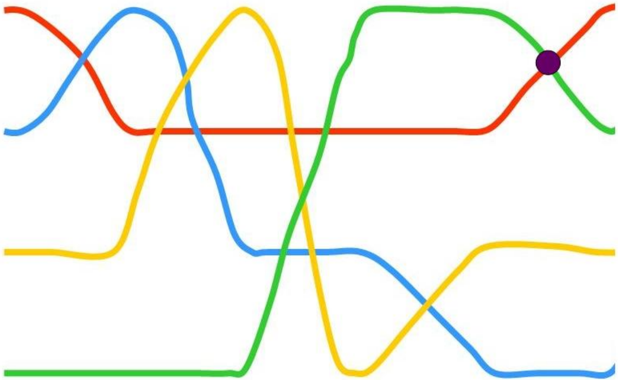
metszéspontok: $1 + 3$



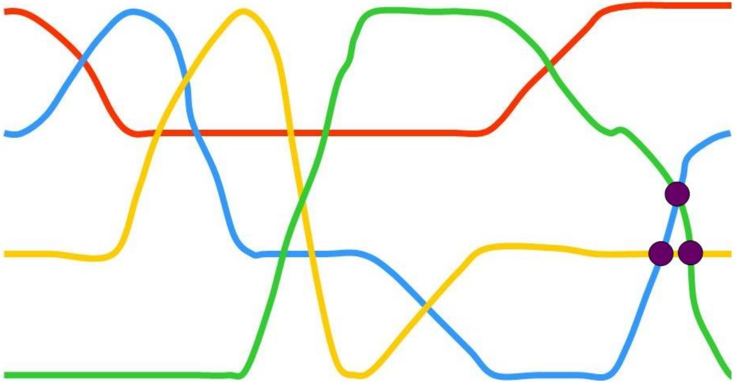
metszéspontok: $1 + 3 + 5$



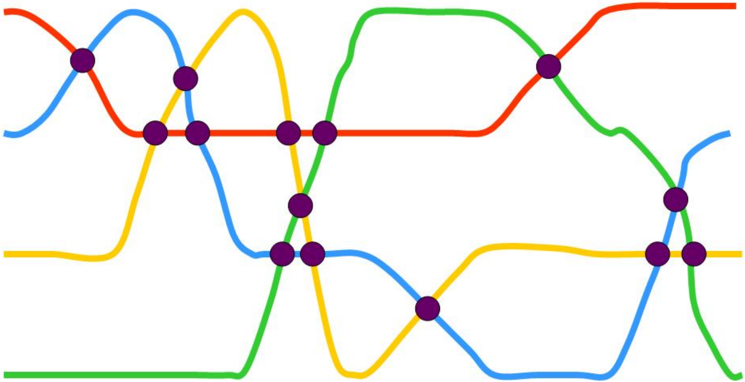
metszéspontok: $1 + 3 + 5 + 1$



metszéspontok: $1 + 3 + 5 + 1 + 1$



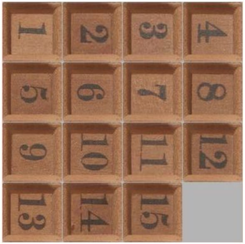
metszéspontok: $1 + 3 + 5 + 1 + 1 + 3 \equiv$ cserék száma $\pmod 2$

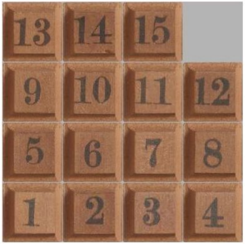


metszéspontok: $0 \equiv$ cserék száma $\pmod 2$


1 2 3 4
5 6 7 8
9 10 11 12
13 15 14


4 8 12
3 7 11 15
2 6 10 14
1 5 9 13


1 2 3 4
5 6 7 8
9 10 11 12
13 14 15


13 14 15
9 10 11 12
5 6 7 8
1 2 3 4


1 2 3
4 5 6 7
8 9 10 11
12 13 14 15


4 3 2 1
8 7 6 5
12 11 10 9
15 14 13

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

$4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$ permutáció

1234 $+$	2134 $-$	3124 $+$	4123 $-$
1243 $-$	2143 $+$	3142 $-$	4132 $+$
1324 $-$	2314 $+$	3214 $-$	4213 $+$
1342 $+$	2341 $-$	3241 $+$	4231 $-$
1423 $+$	2413 $-$	3412 $+$	4312 $-$
1432 $-$	2431 $+$	3421 $-$	4321 $+$

1234 $+$	2134 $-$	3124 $+$	4123 $-$
1243 $-$	2143 $+$	3142 $-$	4132 $+$
1324 $-$	2314 $+$	3214 $-$	4213 $+$
1342 $+$	2341 $-$	3241 $+$	4231 $-$
1423 $+$	2413 $-$	3412 $+$	4312 $-$
1432 $-$	2431 $+$	3421 $-$	4321 $+$

$\frac{4!}{2} = 12$ páros, és ugyanennyi páratlan permutáció

Tizenötös játék

Tizenötös játék

16 kis négyzet:

Tizenötös játék

16 kis négyzet:

$$16! = 20\,922\,789\,888\,000 \text{ permutáció}$$

Tizenötös játék

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$$16! = 20\,922\,789\,888\,000 \text{ permutáció}$$

párosság miatt csak a fele lehetséges:

Tizenötös játék

16 kis négyzet:

$$16! = 20\,922\,789\,888\,000 \text{ permutáció}$$

párosság miatt csak a fele lehetséges:

$$\frac{16!}{2} = \underline{10\,461\,394\,944\,000} \text{ lehetőség}$$

$2\times2\times2$-es bűvös kocka

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8 kis kocka:

$2\times 2\times 2$-es bűvös kocka

8 kis kocka:

$$8! = 40\,320 \text{ permutáció}$$

$2\times2\times2$-es bűvös kocka

8 kis kocka:

$$8! = 40\,320 \text{ permutáció}$$

egy kis kocka 3-féleképpen állhat:

2×2×2-es bűvös kocka

8 kis kocka:

$$8! = 40\,320 \text{ permutáció}$$

egy kis kocka 3-féleképpen állhat:

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^8 = 6561 \text{ orientáció}$$

2×2×2-es bűvös kocka

8 kis kocka:

$$8! = 40\,320 \text{ permutáció}$$

egy kis kocka 3-féleképpen állhat:

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^8 = 6561 \text{ orientáció}$$

az utolsó kis kocka állása kötött:

$2\times2\times2$-es bűvös kocka

8 kis kocka:

$$8! = 40\,320 \text{ permutáció}$$

egy kis kocka 3-féleképpen állhat:

$$3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3 = 3^8 = 6561 \text{ orientáció}$$

az utolsó kis kocka állása kötött:

$$8!\cdot 3^7 = \underline{88\,179\,840} \text{ lehetőség}$$

3×3×3-as bűvös kocka

3$\times$3$\times$3-as bűvös kocka

8 sarokkocka:

3×3×3-as bűvös kocka

8 sarokkocka:

$$8! = 40\,320 \text{ permutáció}, \qquad 3^8 = 6561 \text{ orientáció}$$

$3\times3\times3$-as bűvös kocka

8 sarokkocka:

$$8! = 40\,320 \text{ permutáció}, \qquad 3^8 = 6561 \text{ orientáció}$$

12 élkocka:

$3\times 3\times 3$-as bűvös kocka

8 sarokkocka:

$$8! = 40\,320 \text{ permutáció}, \qquad 3^8 = 6561 \text{ orientáció}$$

12 élkocka:

$$12! = 479\,001\,600 \text{ permutáció}, \qquad 2^{12} = 4096 \text{ orientáció}$$

$3\times3\times3$-as bűvös kocka

8 sarokkocka:

$$8! = 40\,320 \text{ permutáció}, \qquad 3^8 = 6561 \text{ orientáció}$$

12 élkocka:

$$12! = 479\,001\,600 \text{ permutáció}, \qquad 2^{12} = 4096 \text{ orientáció}$$

párosság, utolsó sarok-, ill. élkocka:

3×3×3-as bűvös kocka

8 sarokkocka:

$$8! = 40\,320 \text{ permutáció}, \qquad 3^8 = 6561 \text{ orientáció}$$

12 élkocka:

$$12! = 479\,001\,600 \text{ permutáció}, \qquad 2^{12} = 4096 \text{ orientáció}$$

párosság, utolsó sarok-, ill. élkocka:

$$\frac{8! \cdot 12!}{2} \cdot 3^7 \cdot 2^{11} = \underline{43\,252\,003\,274\,489\,856\,000} \text{ lehetőség}$$