FEJEZETEK A SZÁMELMÉLETBŐL ELŐADÁS

Waldhauser Tamás

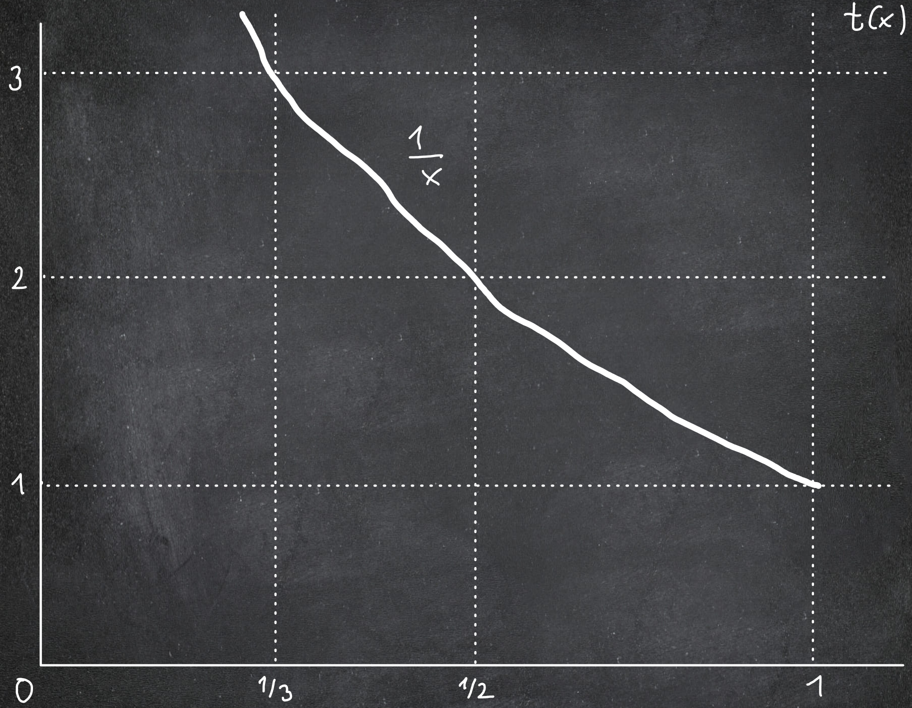
SZTE Bolyai Intézet

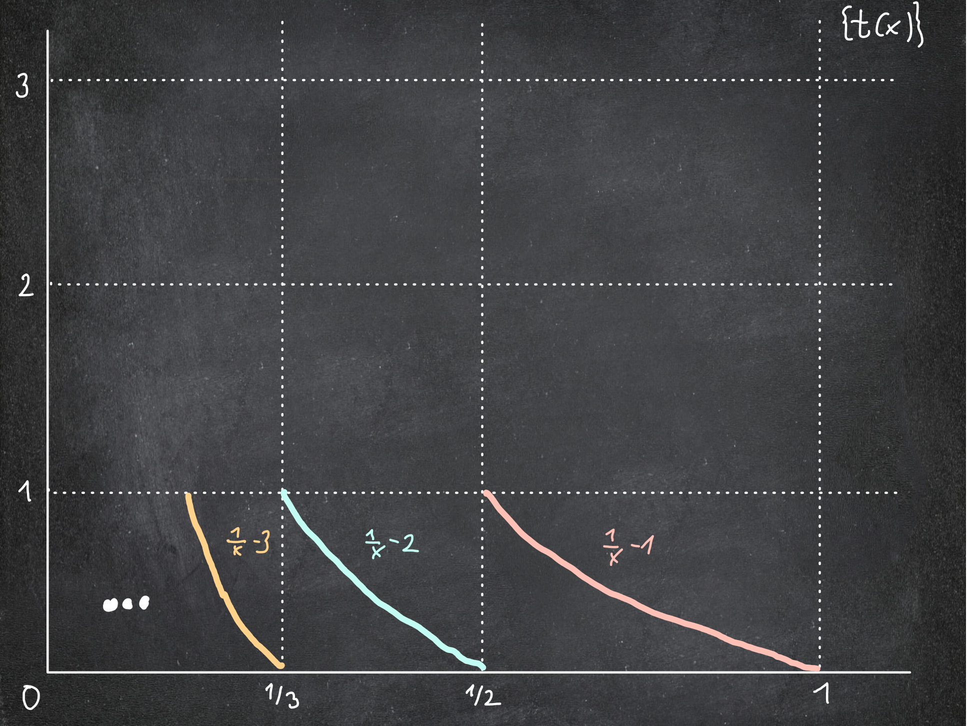
2021. május 18.

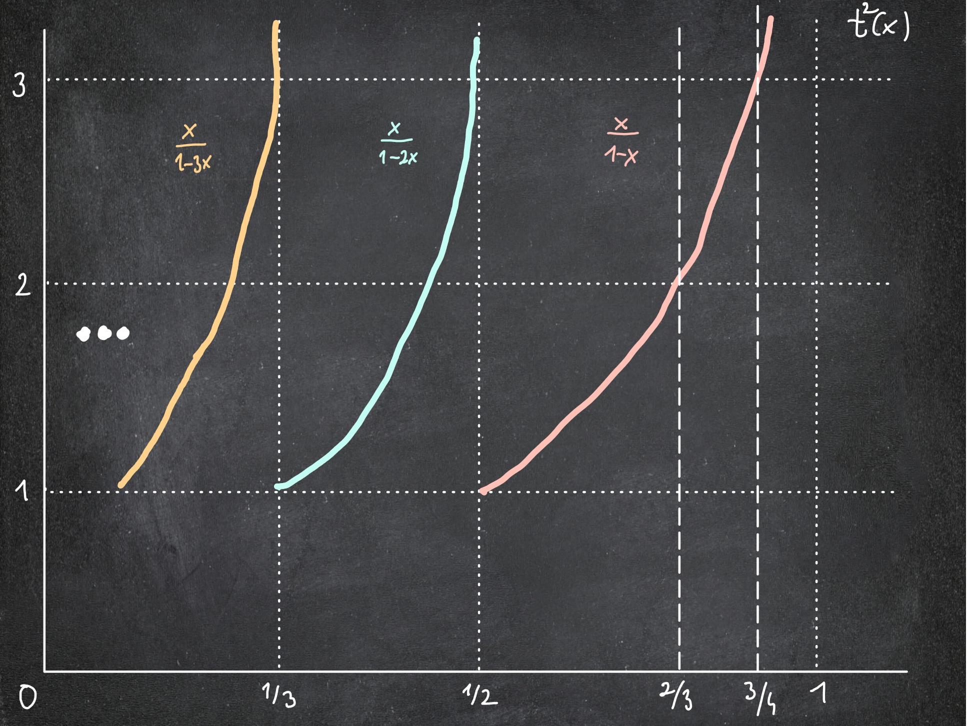
HF 30: $\{x\} > \frac{1}{2} \Rightarrow t^3(x) = t^2(-x)$

$t(x) = t(x+1)$ miatt feltehető, hogy $\frac{1}{2} < x < 1$,

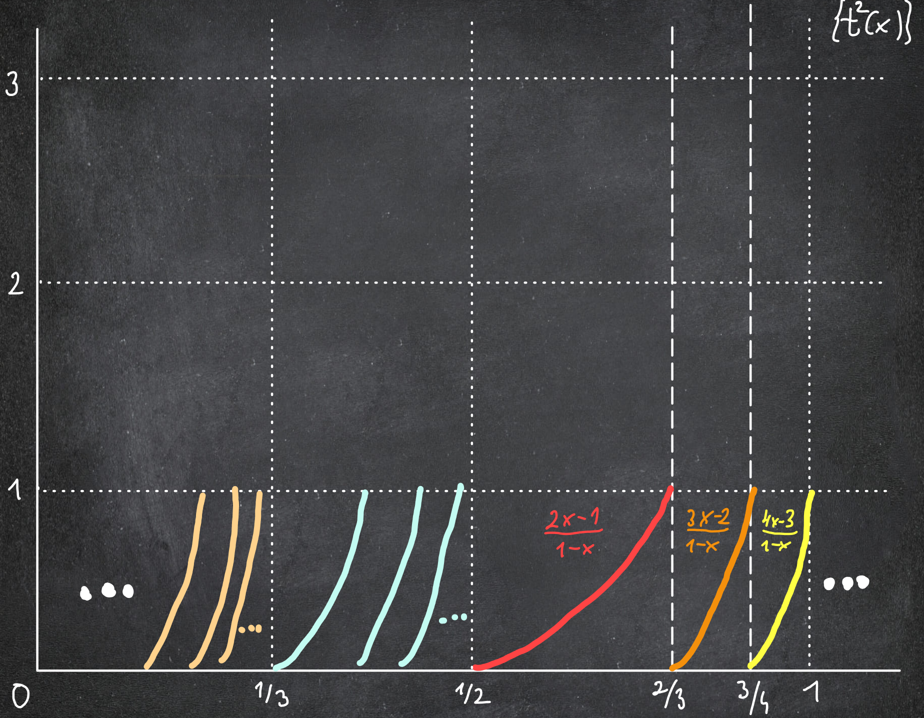
és elég azt belátni, hogy $t^3(x) = t^2(1-x)$


t(x)
3
2
1
0
1/3
1/2
1
1/x

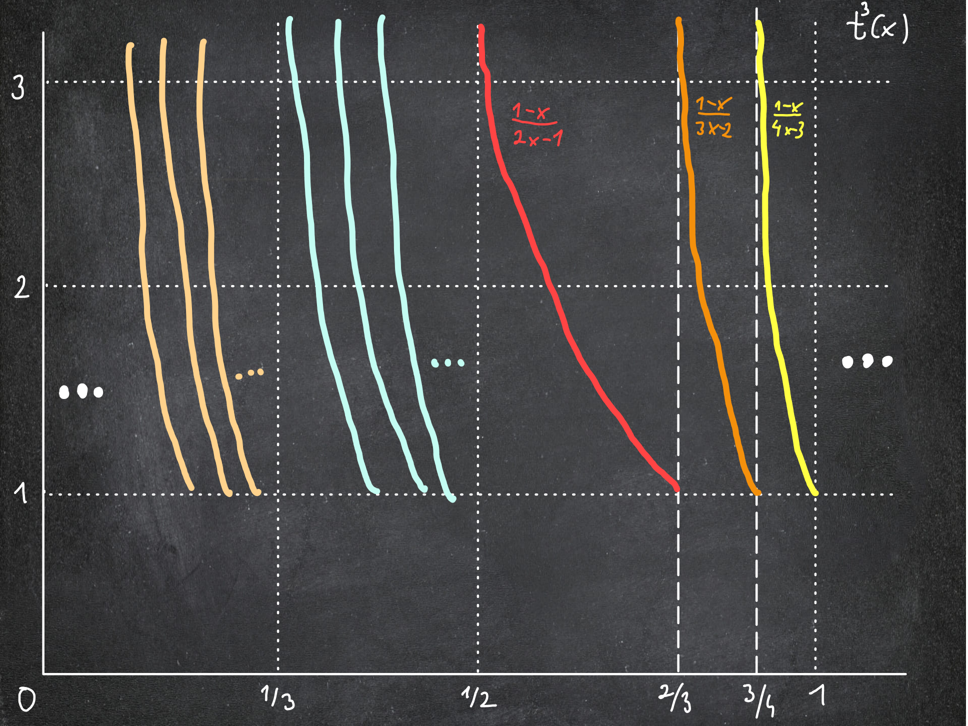

{t(x)}
3
2
1
0
1/3
1/2
1
1/x -3
1/x -2
1/x -1
...



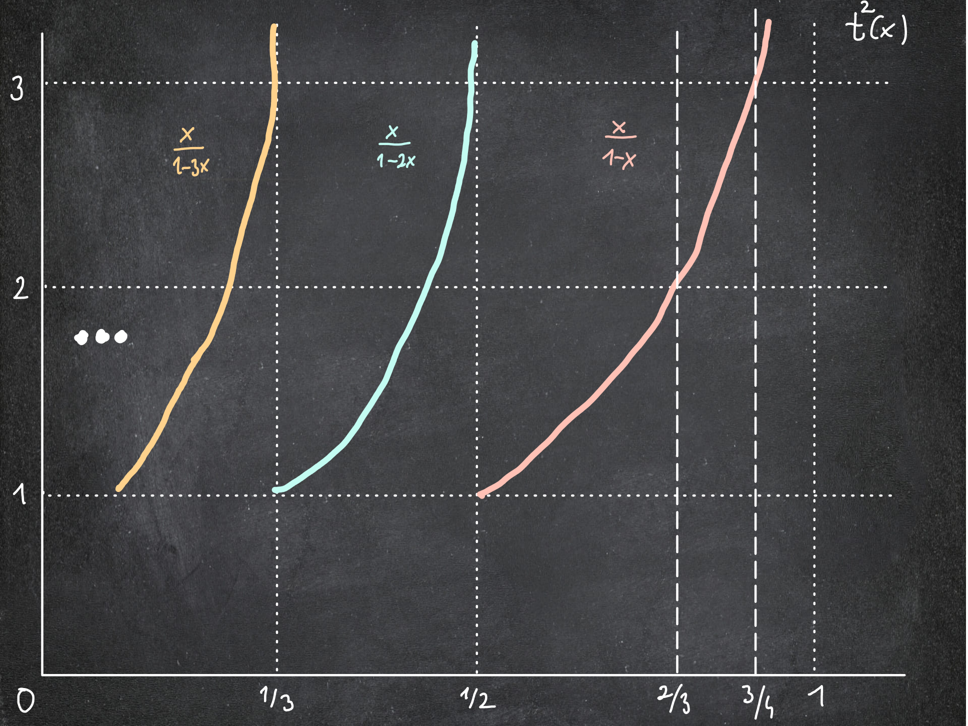
$t^2(x)$
3
2
1
0
$\frac{x}{1-3x}$
$\frac{x}{1-2x}$
$\frac{x}{1-x}$
1/3
1/2
2/3
3/4
1



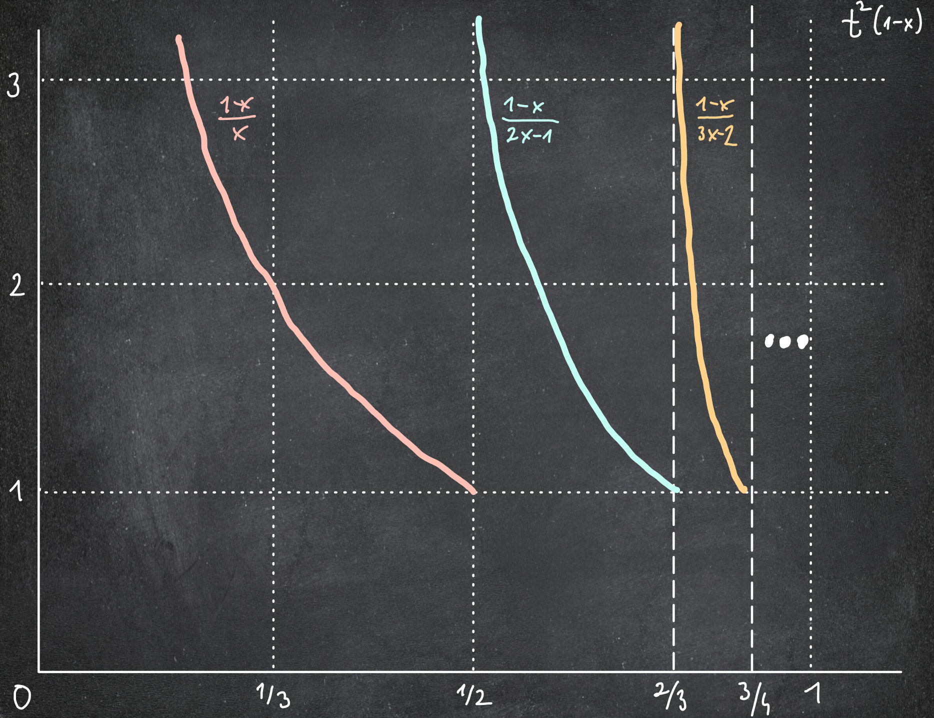
$\{t^2(x)\}$
3
2
1
0
1/3
1/2
2/3
3/4
1
$\frac{2x-1}{1-x}$
$\frac{3x-2}{1-x}$
$\frac{4x-3}{1-x}$



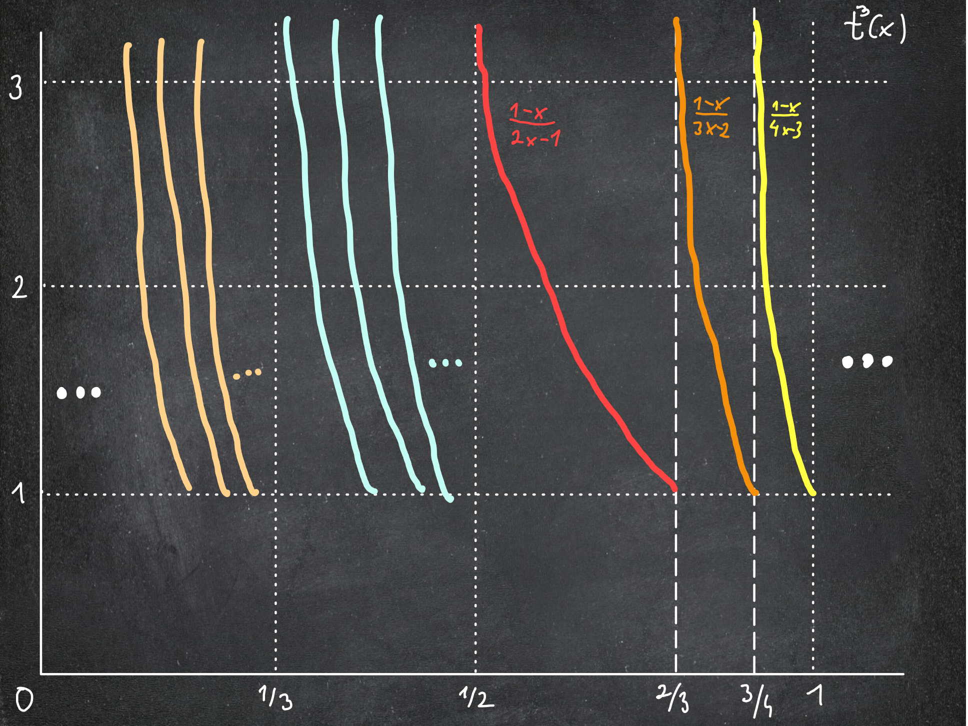
$t^3(x)$
3
2
1
0
1/3
1/2
2/3
3/4
1
$\frac{1-x}{2x-1}$
$\frac{1-x}{3x-2}$
$\frac{1-x}{4x-3}$



$t^2(x)$
3
2
1
0
1/3
1/2
2/3
3/4
1
$\frac{x}{1-3x}$
$\frac{x}{1-2x}$
$\frac{x}{1-x}$



$t^2(1-x)$
3
2
1
0
$\frac{1-x}{x}$
$\frac{1-x}{2x-1}$
$\frac{1-x}{3x-2}$
...
1/3
1/2
2/3
3/4
1


$t^3(x)$
3
2
1
0
1/3
1/2
2/3
3/4
1
$\frac{1-x}{2x-1}$
$\frac{1-x}{3x-2}$
$\frac{1-x}{4x-3}$

$f \in \mathbb{Z}[x]$ főpolinom $\Rightarrow f\left(\frac{1}{\sqrt{2}}\right) \neq 0$

$f = x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$ tfh.

$$f\left(\frac{1}{\sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}\right)^n + a_{n-1}\cdot\left(\frac{1}{\sqrt{2}}\right)^{n-1} + a_{n-2}\cdot\left(\frac{1}{\sqrt{2}}\right)^{n-2} + \dots + a_1\cdot\frac{1}{\sqrt{2}} + a_0 = 0$$

$$1 + a_{n-1}\sqrt{2} + a_{n-2}2 + \dots + a_1\cdot\sqrt{2}^{n-1} + a_0\sqrt{2}^n = 0$$

$$\underbrace{(1 + 2a_{n-2} + 4a_{n-4} + \dots)}_{\text{pt.}\ \lightning} + \underbrace{(a_{n-1} + 2a_{n-3} + 4a_{n-5} + \dots)}_{=0}\sqrt{2} = 0$$

$50_p = 100\%$

$x^2 - 2y^2 = \pm 1$ $\qquad (1+\sqrt{2})^n = x_n + y_n\sqrt{2}$

n	0	1	2	3	4	5	6	7	...
x_n	1	1	3	7	17	41	99	239	...
y_n	0	1	2	5	12	29	70	169	...
	+	−	+	−	+	−	+	−	

$x_{n+1} = 2x_n + x_{n-1}$

$y_{n+1} = 2y_n + y_{n-1}$

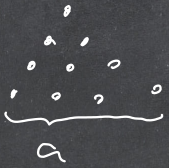
$$\left.\begin{aligned}(1+\sqrt{2})^n &= x_n + y_n\sqrt{2}\\ (1-\sqrt{2})^n &= x_n - y_n\sqrt{2}\end{aligned}\right\} \quad x_n = \frac{1}{2}\Big(\underbrace{(1+\sqrt{2})}_{2,4\ldots}{}^n + \underbrace{(1-\sqrt{2})}_{-0,7\ldots}{}^n\Big)$$

$$= \left\lfloor \frac{1}{2}(1+\sqrt{2})^n \right\rceil$$

$$y_n = \frac{1}{2\sqrt{2}}\Big((1+\sqrt{2})^n - (1-\sqrt{2})^n\Big)$$

$$= \left\lfloor \frac{1}{2\sqrt{2}}(1+\sqrt{2})^n \right\rceil$$

$$\frac{x_n}{y_n} \approx \sqrt{2} \approx \frac{239}{13^2}$$



$$b^2 = a + (a-1) + \dots + 1 = \frac{a(a+1)}{2}$$

$$\frac{a(a+1)}{2} = b^2$$

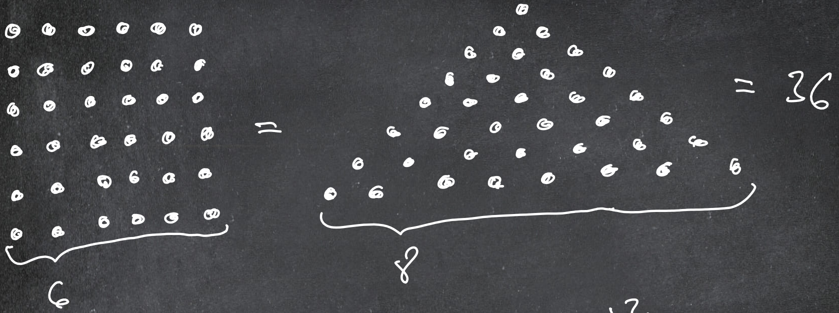
$$a^2 + a = 2b^2$$

$$4a^2 + 4a + 1 = 8b^2 + 1$$

$$(2a+1)^2 = 2\cdot(2b)^2 + 1$$

$$\underbrace{(2a+1)}_{x}^2 - 2\cdot(\underbrace{2b}_{y})^2 = 1$$

$$a = \frac{x_{2n} - 1}{2} \qquad b = \frac{y_{2n}}{2}$$



$$b^2 = \left(\frac{y_{2n}}{2}\right)^2 \approx \left(\frac{1}{2}\,\frac{1}{2\sqrt{2}}\,(1+\sqrt{2})^{2n}\right)^2 = \left[\frac{1}{32}\underbrace{(17+12\sqrt{2})}_{33{,}97}{}^{n}\right]$$

$\parallel$

ST_n

n	1	2	3	4	...
ST_n	1	36	1225	41616	...
	$= 1^2$	$= 6^2$	$= 35^2$	$= 204^2$	

$1 + \ldots + a = (a+1) + \ldots + b$

$a = ?\quad b = ?$

- i.e. 2?? ARKHIMÉDESZ
- i.u. 628 BRAHMAGUPTA SAMASA-módszer

$$N(\alpha\beta) = N(\alpha)\cdot N(\beta)$$

$$x^2 - 23y^2 = 2$$

$x = 5,\ y = 1$ $\quad N(5+\sqrt{23}) = 2$

$$N(48+10\sqrt{23}) = 4$$

$$48^2 - 23\cdot 10^2 = 4 \quad /:4$$

$$24^2 - 23\cdot 5^2 = 1$$

$x = 24,\ y = 5$ $\quad x^2 - 23y^2 = 1$

- 1150 BHASKARA II. CHAKRAVALA-módszer

- 1657 FERMAT

$$x^2 - d y^2 = 1 \qquad d = 61$$

- 1658 BROUNCKER, WALLIS, FRÉNICLE DE BESSY
- 1768 LAGRANGE
- 1849 DIRICHLET σ_K^*

	pozitív Pell		negatív Pell	
d	*x*	*y*	*x*	*y*
2	3	2	1	1
3	2	1		
5	9	4	2	1
6	5	2		
7	8	3		
10	19	6	3	1
11	10	3		
13	649	180	18	5
14	15	4		
15	4	1		
17	33	8	4	1
19	170	39		
21	55	12		
22	197	42		
23	24	5		

	pozitív Pell		negatív Pell	
d	*x*	*y*	*x*	*y*
26	51	10	5	1
29	9 801	1 820	70	13
30	11	2		
31	1 520	273		
33	23	4		
34	35	6		
35	6	1		
37	73	12	6	1
38	37	6		
39	25	4		
41	2 049	320	32	5
42	13	2		
43	3 482	531		
46	24 335	35 88		
47	48	7		

	pozitív Pell		negatív Pell	
d	*x*	*y*	*x*	*y*
51	50	7		
53	66 249	9 100	182	25
55	89	12		
57	151	20		
58	19 603	2 574	99	13
59	530	69		
61	1 766 319 049	226 153 980	29 718	3 805
62	63	8		
65	129	16	8	1
66	65	8		
67	48 842	5 967		
69	7 775	936		
70	251	30		
71	3 480	413		
73	2 281 249	267 000	1 068	125