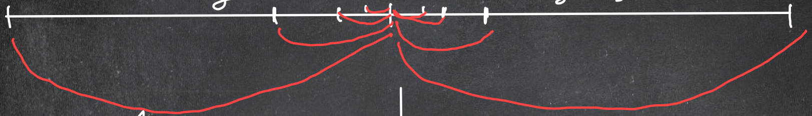
FEJEZETEK A SZÁMELMÉLETBŐL ELŐADÁS

Waldhauser Tamás

SZTE Bolyai Intézet

2021. április 27.

$\frac{a}{b} = \frac{1}{2}$ $\quad \frac{10}{19} = \frac{x}{y} = ?$ $\quad \frac{19}{36} \cdots \frac{9}{17} \cdots \frac{26}{49}$ $\quad ? = \frac{x}{y} = \frac{17}{32}$ $\quad \frac{c}{d} = \frac{8}{15}$



$$\frac{9}{17} - \frac{x}{y} = \frac{1}{17y}$$

$$9y - 17x = 1$$

$$x_0 = 1,\ y_0 = 2$$

$$x = 1 + 9t$$

$$y = 2 + 17t$$

t	0	1	2	...
x	1	10	19	...
y	2	19	36	...

$$\frac{x}{y} - \frac{9}{17} = \frac{1}{17y}$$

$$17x - 9y = 1$$

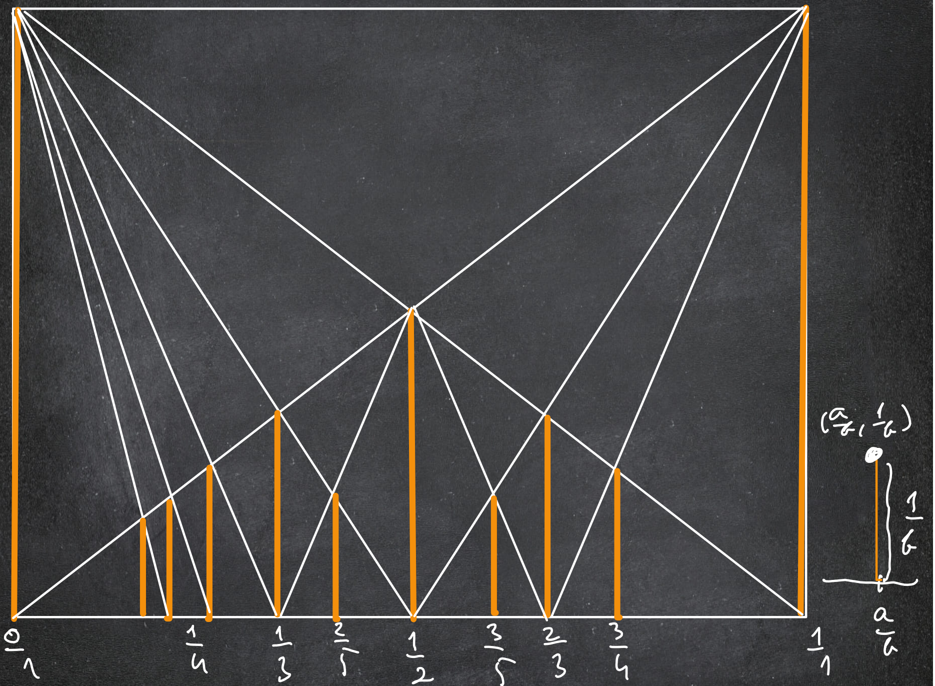
$$x_0 = -1,\ y_0 = -2$$

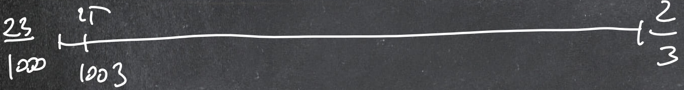
$$x = -1 + 9t$$

$$y = -2 + 17t$$

t	1	2	3	...
x	8	17	26	...
y	15	32	49	...

0/1
1/5
1/4
2/7
1/3
3/8
2/5
3/7
1/2
4/7
3/5
5/8
2/3
5/7
3/4
4/5
1/1


$\left(\frac{a}{b}, \frac{1}{b}\right)$
$\frac{1}{b}$
$\frac{0}{1}$
$\frac{1}{4}$
$\frac{1}{3}$
$\frac{2}{5}$
$\frac{1}{2}$
$\frac{3}{5}$
$\frac{2}{3}$
$\frac{3}{4}$
$\frac{1}{1}$
$\frac{a}{b}$



véges utak a SB fában $\Longleftrightarrow$ poz. rac. számok

végtelen utak a SB fában $\Longleftrightarrow$ poz. irrac. számok

$\uparrow$

amiben ∞ sok

fordulópont van

$$\frac{p}{q} = \frac{a+c}{b+d} \qquad \left[\frac{a}{b}, \frac{c}{d}\right] \qquad \frac{c}{d} - \frac{a}{b} = \frac{1}{bd} \qquad cb - ad = 1 \qquad \begin{pmatrix} c & a \\ d & b \end{pmatrix} \rightsquigarrow \frac{cx+a}{dx+b}$$

$$\swarrow L \qquad \searrow R$$

$$\rightsquigarrow \quad L: \left[\frac{a}{b}, \frac{a+c}{b+d}\right] \qquad R: \left[\frac{a+c}{b+d}, \frac{c}{d}\right]$$

$$\rightsquigarrow \quad L: \begin{pmatrix} a+c & a \\ b+d & b \end{pmatrix} \qquad R: \begin{pmatrix} c & a+c \\ d & b+d \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} c & a \\ d & b \end{pmatrix}}_{A} \overbrace{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}}^{L} = \underbrace{\begin{pmatrix} a+c & a \\ b+d & b \end{pmatrix}}_{AL}$$

$$\underbrace{\begin{pmatrix} c & a \\ d & b \end{pmatrix}}_{A} \overbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}^{R} = \underbrace{\begin{pmatrix} c & a+c \\ d & b+d \end{pmatrix}}_{AR}$$

$$A \xrightarrow{L} AL \qquad A \xrightarrow{R} AR$$

$$SL_2(\mathbb{Z}) = [L, R]$$

$$E \xrightarrow{L} L, \quad E \xrightarrow{R} R; \qquad L \to L^2, \; L \to LR; \qquad R \to RL, \; R \to R^2$$

$$\frac{0}{1} \qquad \frac{1}{0} \qquad \frac{1}{1}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$$

$$\begin{tikzcd}[column sep=small] & A \arrow[dl, "L"'] \arrow[dr, "R"] & \\ AL & & AR \end{tikzcd} \rightsquigarrow \begin{tikzcd}[column sep=small] & A \circ x \arrow[dl, no head] \arrow[dr, no head] & \\ AL \circ x & & AR \circ x \end{tikzcd} \rightsquigarrow \begin{tikzcd}[column sep=small] & A \circ 1 \arrow[dl, no head] \arrow[dr, no head] & \\ AL \circ 1 & & AR \circ X \end{tikzcd}$$

$$R \circ X = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \frac{1 \cdot x + 1}{0 \cdot x + 1} = x + 1$$

$$\boxed{R^a \circ X = x + a}$$

$$L \circ X = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \circ x = \frac{1 \cdot x + 0}{1 \cdot x + 1} = \frac{x}{x+1} = \frac{1}{1 + \frac{1}{x}}$$

$$L^2 \circ X = \frac{1}{2 + \frac{1}{x}} \qquad L^3 \circ X = \frac{1}{3 + \frac{1}{x}} \qquad \boxed{L^a \circ X = \frac{1}{a + \frac{1}{x}}}$$

$$R^{a_0} L^{a_1} R^{a_2} \circ X = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + x}}$$

$$x = 1 = \frac{1}{x}$$

$$R^{a_0} L^{a_1} R^{a_2} L^{a_3} \circ X = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + 1/x}}}$$

$$R^{a_0} L^{a_1} \cdots ?^{a_n} \cdot 1 = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n + 1}}}}$$

$\uparrow$ L v. R

$$R^{a_0} L^{a_1} \cdots R^{a_n} L^{a_{n+1}} \cdots$$

$\uparrow$ fordulópont

Fordulópont előtt: $R^{a_0} L^{a_1} \cdots R^{a_n - 1} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n}}}}$

$= [a_0; a_1, \dots, a_n]$

véges lánctört

$a_0 \in \cancel{\mathbb{N}_0}\ \mathbb{Z}$, $a_1, \dots, a_n \in \mathbb{N}$, $a_n > 1$

$$\alpha = \frac{101}{23} = \underbrace{a_0}_{\lfloor \alpha \rfloor \in \mathbb{Z}} + \underbrace{\frac{1}{\underbrace{a_1 + \ldots}_{>1}}}_{\{\alpha\}} \quad < 1, \ \in [0,1)$$

$$= [4;2,1,1,4] = \cancel{[4;1,1,3,1]}$$

$$0{,}23 = 0{,}\cancel{2295}\ldots -$$

$$\frac{101}{23} = 4 + \frac{9}{23} = 4 + \frac{1}{\frac{23}{9}} = 4 + \frac{1}{2 + \frac{5}{9}} = 4 + \frac{1}{2 + \frac{1}{\frac{9}{5}}} = 4 + \frac{1}{2 + \frac{1}{1 + \frac{4}{5}}} =$$

$$= 4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\frac{5}{4}}}} = 4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4}}}} = 4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1}}}}}$$

$$101 = 4 \cdot 23 + 9 \rightarrow \frac{101}{23} = 4 + \frac{9}{23} = 4 + \frac{1}{2 + \frac{5}{9}} = \dots$$

$$23 = 2 \cdot 9 + 5 \rightarrow \frac{23}{9} = 2 + \frac{5}{9}$$

$$9 = 1 \cdot 5 + 4 \rightarrow \dots$$

$$5 = 1 \cdot 4 + 1 \rightarrow \dots$$

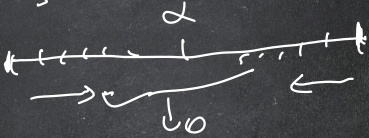
$$4 = 4 \cdot 1 + 0$$

Végtelen lánctört: $[a_0; a_1, a_2, \dots]$ $\quad a_0 \in \mathbb{Z},\ a_1, \dots \in \mathbb{N}$

$$[a_0, a_1, a_2, \dots, a_n] = \frac{p_n}{q_n}$$ n-edik konvergens

$$\lim_{n \to \infty} \frac{p_n}{q_n} = \alpha = [a_0; a_1, a_2, \dots]$$

$\alpha \in \mathbb{R} \setminus \mathbb{Q}$



pontos ⊆ ford. előállít. ⊆ jól ⊆ FSB
||
konvergensek

$$\alpha = [a_0; a_1 \dots] \Rightarrow a_0 = \lfloor \alpha \rfloor$$

$$\frac{1}{[a_1, a_2, \dots]} = \{\alpha\} \qquad [a_1, \dots] = \frac{1}{\{\alpha\}}$$

$$\frac{101}{23} = [4; 2, 1, 1, 4]$$

$$\pi = [3; 7, 15, 1 \mid 292, 1, 1, 1, 2, \dots] \approx 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cancel{\cfrac{1}{292 + \dots}}}}} = =$$

$$= [3; 7, \cancel{15}, 1] = [3; 7, 16] = \frac{355}{113} = \underline{3{,}141592}5$$

$\boxed{HF27}$: $\quad -\pi = [\ldots \quad]$ (mi lesz π-ben?)

$\boxed{HF28.}$ $\begin{cases} e = [\ \ldots &] \\ -e = [\ \ldots &] \end{cases}$ (Szabályszerűség?)

$\boxed{HF29.}$

$t : \mathbb{R} \setminus \mathbb{Z} \to \mathbb{R}, \quad x \mapsto \dfrac{1}{\{x\}}$

Ábrázoljuk $t(x)$-et és $t(t(x))$-et

$$t(t(x)) = \frac{1}{\left\{ \frac{1}{\{x\}} \right\}}$$