$$[B] = \{a^{3k}, a^{3k+2}t \mid k \in \mathbb{Z}\}$$
$$= \{id, a^3, a^6, a^9, at, a^4t, a^7t, a^{10}t\}$$

$\| S \|$

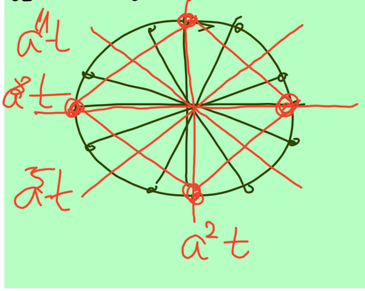
D_4

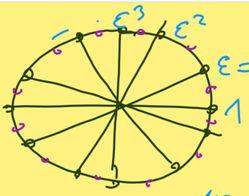
$a^{3l} \cdot a^{3l} = a^{3(2l)}$

$a^{3l} \cdot a^{3l+2}t = a^{3(2l+1)+2}t$

$a^{3l+2}t \cdot a^{3l} = a^{3(l-l)+2}t$

$a^{3l+2}t \cdot a^{3l+2}t = a^{3(l-l)}$





$T_{12} = \{1, \varepsilon, \varepsilon^2, \dots, \varepsilon^{11}\} \cong$

$\varphi \downarrow \cong$

$\mathbb{Z}_{12} = \{\bar{0}, \bar{1}, \bar{2}, \dots, \bar{11}\}$

$\mathbb{Z}_{12}$-ben $[\bar{2}, \bar{3}] = \{\bar{1}, \bar{2}, \dots \bar{p}\}$

$= \mathbb{Z}_{12}$

$[\varepsilon^2, \varepsilon^3] = [\varepsilon^2\varphi, \varepsilon^3\varphi]\varphi^{-1}$

$= \mathbb{Z}_{12}\varphi^{-1} = T_{12}$

$\mathbb{Z}_{24} = [\{\bar{4}, \bar{6}\}] = \{\bar{0}, \bar{2}, \dots, \bar{22}\}$

$[(1,1,-2), (1,-2,1), (-2,1,1)] = [B]$

$(a,b,c) + (a',b',c') = (a+a', b+b', c+c')$

$a \equiv b \equiv c \quad a' \equiv b' \equiv c' \Rightarrow a+a' \equiv b+b' \equiv c+c'$

$a+b+c \equiv 0 \quad a'+b'+c' \equiv 0 \Rightarrow (a+a')+(b+b')+(c+c') \equiv 0$

$[B] \subseteq \{(a,b,c) \mid a+b+c \equiv 0 \text{ és } a \equiv b \equiv c\}$

$\supseteq$ ✓ $x(1,1,-2) + y(1,-2,1) = (a,b,c)$

$x+y=a$
$x-2y=b$
$-2x+y=c$

$y = \frac{a-b}{3} \in \mathbb{Z}$

$x = \frac{2a+b}{3} \in \mathbb{Z}$

$2x - y = a+b \equiv -c$

$[(-1,-1,2), (-1,2,-1), (2,-1,-1)] = [B]$

b_1 b_2 b_3

$b_1 + b_2 = -b_3$

$g = (368)(46)$

$g^k \neq (368)^k(46)^k = id \Leftrightarrow 3 \mid k \text{ és } 2 \mid k \Leftrightarrow 6 \mid k$

$\Rightarrow o(g) = 6$

$g = (3468)$

$g^k = id \Leftrightarrow 4 \mid k$

$o(g) = 4$