

## Részcsoporthálók

# Részcsoportháló

Jelölje  $\text{Sub}(G)$  a  $G$  csoport részcsoportjainak halmazát. Például

$$\text{Sub}(\mathbb{Z}_4) = \{\{\bar{0}\}, \{\bar{0}, \bar{2}\}, \{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}\}.$$

A  $(\text{Sub}(G); \subseteq)$  részbenrendezett halmaz háló, amelyet  $G$  **részcsoporthálójának** nevezünk. A hálóműveletek:

$H \wedge K = H \cap K$  (a legbővebb részcsoport, ami része  $H$ -nak is és  $K$ -nak is);

$H \vee K = [H \cup K]$  (a legszűkebb részcsoport, ami tartalmazza  $H$ -t is és  $K$ -t is).

## $\mathbb{Z}_{12}$ részcsoporthálója

Minden részcsoport ciklikus:

- ▶  $[1] = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \bar{10}, \bar{11}\}$
- ▶  $[2] = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \bar{10}\}$
- ▶  $[3] = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}$
- ▶  $[4] = \{\bar{0}, \bar{4}, \bar{8}\}$
- ▶  $[5] = \{\bar{0}, \bar{5}, \bar{10}, \bar{3}, \bar{8}, \bar{1}, \bar{6}, \bar{11}, \bar{4}, \bar{9}, \bar{2}, \bar{7}\}$
- ▶  $[6] = \{\bar{0}, \bar{6}\}$
- ▶  $[7] = \{\bar{0}, \bar{7}, \bar{2}, \bar{9}, \bar{4}, \bar{11}, \bar{6}, \bar{1}, \bar{8}, \bar{3}, \bar{10}, \bar{5}\}$
- ▶  $[8] = \{\bar{0}, \bar{8}, \bar{4}\}$
- ▶  $[9] = \{\bar{0}, \bar{9}, \bar{6}, \bar{3}\}$
- ▶  $[10] = \{\bar{0}, \bar{10}, \bar{8}, \bar{6}, \bar{4}, \bar{2}\}$
- ▶  $[11] = \{\bar{0}, \bar{11}, \bar{10}, \bar{9}, \bar{8}, \bar{7}, \bar{6}, \bar{5}, \bar{4}, \bar{3}, \bar{2}, \bar{1}\}$
- ▶  $[0] = \{\bar{0}\}$

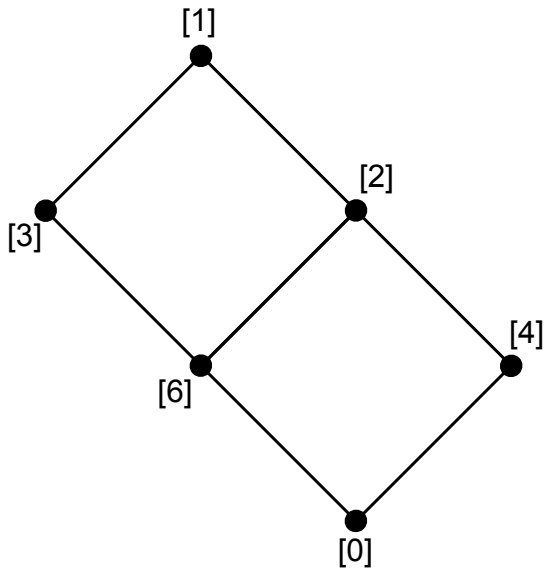
## $\mathbb{Z}_{12}$ részcsoporthálója

Minden részcsoport ciklikus:

- ▶  $[1] = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \bar{10}, \bar{11}\} = [5] = [7] = [11]$
- ▶  $[2] = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \bar{10}\} = [10]$
- ▶  $[3] = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} = [9]$
- ▶  $[4] = \{\bar{0}, \bar{4}, \bar{8}\} = [8]$
- ▶  $[6] = \{\bar{0}, \bar{6}\}$
- ▶  $[0] = \{\bar{0}\}$

# $\mathbb{Z}_{12}$ részcsoporthálója

A részcsoportháló:



## $D_4$ részcsoporthálója

$$D_4 = \{\text{id}, a, a^2, a^3, t, at, a^2t, a^3t\}$$

A ciklikus részcsoportok:

- ▶  $[\text{id}] = \{\text{id}\}$
- ▶  $[t] = \{\text{id}, t\}$
- ▶  $[at] = \{\text{id}, at\}$
- ▶  $[a^2t] = \{\text{id}, a^2t\}$
- ▶  $[a^3t] = \{\text{id}, a^3t\}$
- ▶  $[a] = \{\text{id}, a, a^2, a^3\} = [a^3]$
- ▶  $[a^2] = \{\text{id}, a^2\}$

## $D_4$ részcsoporthálója

$$D_4 = \{\text{id}, a, a^2, a^3, t, at, a^2t, a^3t\}$$

A ciklikus részcsoportok:

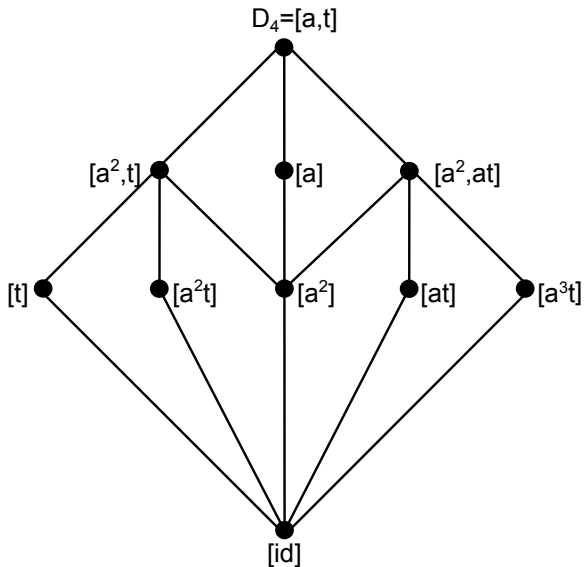
- ▶  $[\text{id}] = \{\text{id}\}$
- ▶  $[t] = \{\text{id}, t\}$
- ▶  $[at] = \{\text{id}, at\}$
- ▶  $[a^2t] = \{\text{id}, a^2t\}$
- ▶  $[a^3t] = \{\text{id}, a^3t\}$
- ▶  $[a^2] = \{\text{id}, a^2\}$
- ▶  $[a] = \{\text{id}, a, a^2, a^3\} = [a^3]$

További részcsoportok:

- ▶  $[a^2, t] = \{\text{id}, a^2, t, a^2t\} = [a^2, a^2t] = [t, a^2t] \cong V$
- ▶  $[a^2, at] = \{\text{id}, a^2, at, a^3t\} = [a^2, a^3t] = [at, a^3t] \cong V$
- ▶  $[a, t] = D_4$

# $D_4$ részcsoporthálója

A részcsoportháló:





## $\mathbb{Z}_{21}^*$ részcsoporthálója

$$\mathbb{Z}_{21}^* = \{\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{8}, \bar{10}, \bar{11}, \bar{13}, \bar{16}, \bar{17}, \bar{19}, \bar{20}\}$$

A ciklikus részcsoportok:

- ▶  $[\bar{1}] = \{\bar{1}\}$
- ▶  $[\bar{2}] = \{\bar{1}, \bar{2}, \bar{4}, \bar{8}, \bar{16}, \bar{11}\} = [\bar{11}]$
- ▶  $[\bar{4}] = \{\bar{1}, \bar{4}, \bar{16}\} = [\bar{16}]$
- ▶  $[\bar{5}] = \{\bar{1}, \bar{5}, \bar{4}, \bar{20}, \bar{16}, \bar{17}\} = [\bar{17}]$
- ▶  $[\bar{8}] = \{\bar{1}, \bar{8}\}$
- ▶  $[\bar{10}] = \{\bar{1}, \bar{10}, \bar{16}, \bar{13}, \bar{4}, \bar{19}\} = [\bar{19}]$
- ▶  $[\bar{13}] = \{\bar{1}, \bar{13}\}$
- ▶  $[\bar{20}] = \{\bar{1}, \bar{20}\}$

## $\mathbb{Z}_{21}^*$ részcsoporthálója

$$\mathbb{Z}_{21}^* = \{\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{8}, \bar{10}, \bar{11}, \bar{13}, \bar{16}, \bar{17}, \bar{19}, \bar{20}\}$$

A ciklikus részcsoportok:

- ▶  $[\bar{1}] = \{\bar{1}\}$
- ▶  $[-\bar{1}] = \{\bar{1}, -\bar{1}\}$
- ▶  $[\bar{8}] = \{\bar{1}, \bar{8}\}$
- ▶  $[-\bar{8}] = \{\bar{1}, -\bar{8}\}$
- ▶  $[\bar{4}] = \{\bar{1}, \bar{4}, -\bar{5}\}$
- ▶  $[\bar{2}] = \{\bar{1}, \bar{2}, \bar{4}, -\bar{5}, \bar{8}, -\bar{10}\}$
- ▶  $[-\bar{2}] = \{\bar{1}, -\bar{2}, \bar{4}, -\bar{5}, -\bar{8}, \bar{10}\}$
- ▶  $[-\bar{4}] = \{\bar{1}, -\bar{1}, \bar{4}, -\bar{4}, \bar{5}, -\bar{5}\}$

További részcsoportok:

- ▶  $[-\bar{1}, \bar{8}] = \{\bar{1}, -\bar{1}, \bar{8}, -\bar{8}\} \cong V$
- ▶  $\mathbb{Z}_{21}^*$

# $\mathbb{Z}_{21}^*$ részcsoporthálója

A részcsoportháló:

