MINOR POSETS OF FUNCTIONS AS QUOTIENTS OF PARTITION LATTICES

(electronic supplementary material for the paper in Order)

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Let c be an admissible coloring of Π_n, and let $\alpha \in \Pi_n$. We claimed in Remark 22 in the article that changing all colors outside of $\downarrow\alpha$ to $c(\alpha)$, the resulting new coloring will not be admissible in general. To justify this claim, consider the coloring shown in Figure 1. This is an admissible coloring of Π_5 using 39 colors (note that the 29 gray pentagons have all different colors). The partition $\alpha = \{\{1,2,3\},\{4,5\}\}$ is highlighted by a thick red outline in the figure. Now if we change all colors outside the principal ideal $\downarrow\alpha$ to yellow (see Figure 2), then this new coloring is not admissible anymore. Indeed, in the modified coloring one of the two red partitions has an orange upper cover, while the other does not, showing that the two red partitions are not $\sim$-related.

We also provide an interactive version of this figure[1] (see Online Resource 2), which can be opened by Wolfram Mathematica or by the freely available Wolfram CDF Player. This file has several features that make it easier to investigate the colorings (in particular, to verify the above claims).

- Hover the mouse cursor over a pentagon to see a tooltip showing the corresponding partition and the number of its color.
- Choose between the original and the modified coloring from the drop-down menu.
- Check the checkbox "show principal ideals" and click on a pentagon to highlight the edges in the principal ideal generated by the corresponding partition. (This is helpful for verifying that the coloring is changed only outside of $\downarrow\alpha$.)
- Check the checkbox "show principal filters" and click on a pentagon to highlight the edges in the principal filter generated by the corresponding partition. (This is helpful for verifying that the original coloring is admissible and the modified coloring is not.)

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[1] This is based on the demonstration "Set Partition Refinement Lattice" by Robert Dickau.

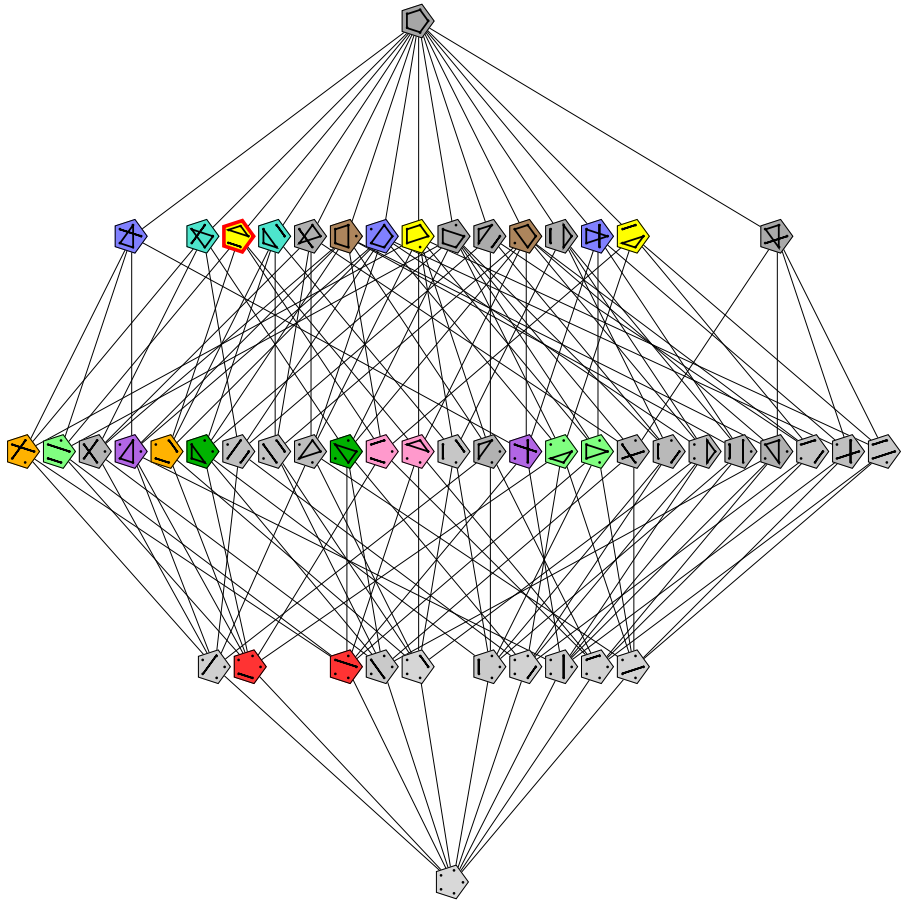

FIGURE 1. The original (admissible) coloring of Π_5

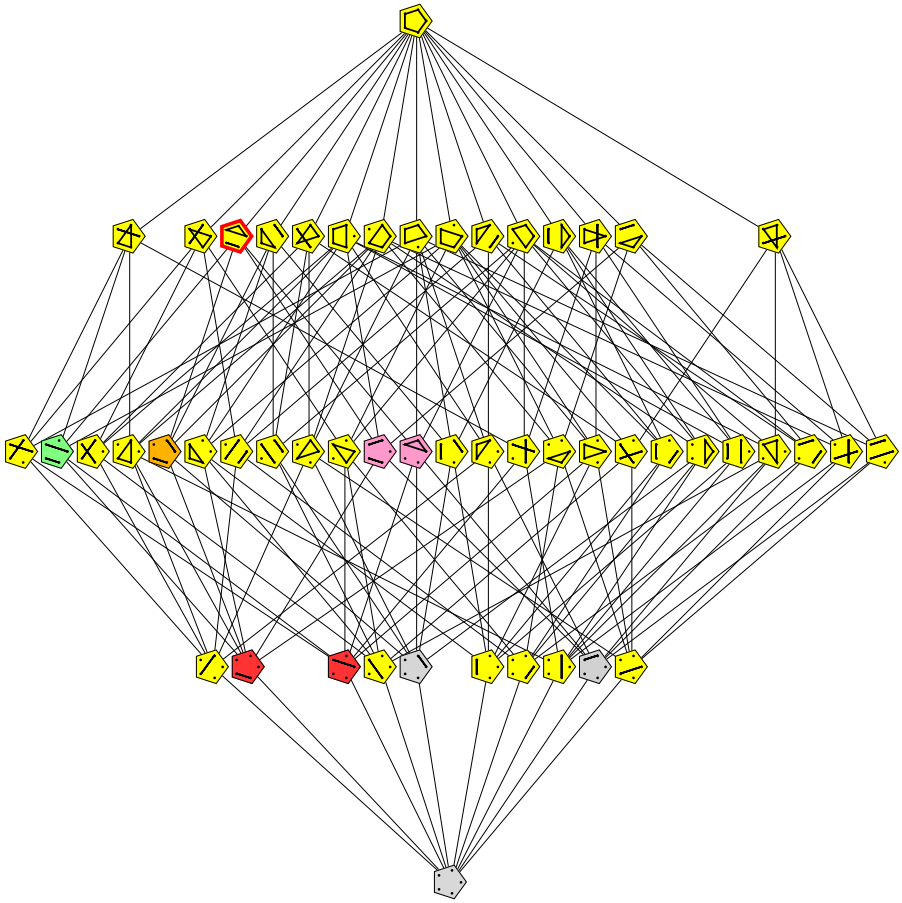

FIGURE 2. The modified (not admissible) coloring of Π_5