```
> restart:
  with(linalg):

  A:=matrix(3,3,[-4,-3,-2,-1,0,1,2,3,4]);

  B:=evalm(transpose(A)&*A);

  P:=charpoly(B,t);
  eigenvectors(B);

  lambda:=vector(3,[54,6,0]);
  x:=array(1..3);
  x[1]:=vector(3,[1,1,1]);
  x[2]:=vector(3,[1,0,-1]);
  x[3]:=vector(3,[1,-2,1]);
  xnorm:=array(1..3):
  for k from 1 to 3 do:
  xnorm[k]:=0:
  for i from 1 to 3 do:
  xnorm[k]:=xnorm[k]+x[k][i]^2:
  od: #i
  xnorm[k]:=sqrt(xnorm[k]):
  od: #k
  print("Normak",xnorm[1],xnorm[2],xnorm[3]);
  print("E oszlopai");

  E:=matrix(3,3):
  e:=array(1..3):
  for j from 1 to 3 do:
  e[j]:=matrix(3,1,[
  1/xnorm[j]*x[j][1],1/xnorm[j]*x[j][2],1/xnorm[j]*x[j][3] ]):
  print(e[j]):
  od:
  for j from 1 to 3 do:
  for i from 1 to 3 do:
  E[i,j]:=e[j][i,1]:
  od:od:
  print("E=",evalm(E));


  print("ELLENORZES",
  evalm(B-E&*diag(lambda[1],lambda[2],lambda[3])&*transpose(E)));


  Lambda:=diag(sqrt(lambda[1]),sqrt(lambda[2]),sqrt(lambda[3]));
  Lambdainv:=diag(1/sqrt(lambda[1]),1/sqrt(lambda[2]),0);
```

```
F0:=evalm(A&*E&*Lambdainv);

print("ELLENORZES",
evalm(A-F0&*Lambda&*transpose(E)));

Vectprmat:=matrix(3,3,[
F0[1,1],F0[1,2],ii, F0[2,1],F0[2,2],jj, F0[3,1],F0[3,2],kk]);
Vectpr:=det(Vectprmat);

f3:=matrix(3,1,[diff(Vectpr,ii),diff(Vectpr,jj),diff(Vectpr,kk)]
);
F:=matrix(3,3,[
F0[1,1],F0[1,2],f3[1,1], F0[2,1],F0[2,2],f3[2,1],
F0[3,1],F0[3,2],f3[3,1]]);
print("SVD felbontas");
print(evalm(A),"=",evalm(F),"*",evalm(Lambda),"*",evalm(transpos
e(E)));
```

```
Warning, new definition for norm
Warning, new definition for trace
```

$$A := \begin{bmatrix} -4 & -3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$B := \begin{bmatrix} 21 & 18 & 15 \\ 18 & 18 & 18 \\ 15 & 18 & 21 \end{bmatrix}$$

$$P := t^3 - 60\,t^2 + 324\,t$$

$$[0, 1, \{[1, -2, 1]\}], [6, 1, \{[-1, 0, 1]\}], [54, 1, \{[1, 1, 1]\}]$$

$$\lambda := [54, 6, 0]$$

$$x := \mathrm{array}(1 .. 3, [\ ])$$

$$x_1 := [1, 1, 1]$$

$$x_2 := [1, 0, -1]$$

$$x_3 := [1, -2, 1]$$

$$\text{"Normak"}, \sqrt{3}, \sqrt{2}, \sqrt{6}$$

"E oszlopai"

$$\begin{bmatrix} \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ -\frac{1}{2}\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{6}\sqrt{6} \\ -\frac{1}{3}\sqrt{6} \\ \frac{1}{6}\sqrt{6} \end{bmatrix}$$

$$\text{"E="}, \begin{bmatrix} \frac{1}{3}\sqrt{3} & \frac{1}{2}\sqrt{2} & \frac{1}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{3} & 0 & -\frac{1}{3}\sqrt{6} \\ \frac{1}{3}\sqrt{3} & -\frac{1}{2}\sqrt{2} & \frac{1}{6}\sqrt{6} \end{bmatrix}$$

$$\text{"ELLENORZES"}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Lambda := \begin{bmatrix} 3\sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Lambdainv := \begin{bmatrix} \frac{1}{18}\sqrt{6} & 0 & 0 \\ 0 & \frac{1}{6}\sqrt{6} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F0 := \begin{bmatrix} -\frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & 0 \\ 0 & -\frac{1}{6}\sqrt{2}\sqrt{6} & 0 \\ \frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & 0 \end{bmatrix}$$

$$\text{"ELLENORZES"}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Vectprmat := \begin{bmatrix} -\frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & ii \\ 0 & -\frac{1}{6}\sqrt{2}\sqrt{6} & jj \\ \frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & kk \end{bmatrix}$$

$$Vectpr := \frac{1}{6}\sqrt{3}\sqrt{2}\,kk - \frac{1}{3}\sqrt{3}\,jj\sqrt{2} + \frac{1}{6}\sqrt{3}\,ii\sqrt{2}$$

$$f3 := \begin{bmatrix} \frac{1}{6}\sqrt{3}\sqrt{2} \\ -\frac{1}{3}\sqrt{3}\sqrt{2} \\ \frac{1}{6}\sqrt{3}\sqrt{2} \end{bmatrix}$$

$$F := \begin{bmatrix} -\frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & \frac{1}{6}\sqrt{3}\sqrt{2} \\ 0 & -\frac{1}{6}\sqrt{2}\sqrt{6} & -\frac{1}{3}\sqrt{3}\sqrt{2} \\ \frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & \frac{1}{6}\sqrt{3}\sqrt{2} \end{bmatrix}$$

"SVD felbontas"

$$\begin{bmatrix} -4 & -3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}, \text{"="}, \begin{bmatrix} -\frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & \frac{1}{6}\sqrt{3}\sqrt{2} \\ 0 & -\frac{1}{6}\sqrt{2}\sqrt{6} & -\frac{1}{3}\sqrt{3}\sqrt{2} \\ \frac{1}{6}\sqrt{3}\sqrt{6} & -\frac{1}{6}\sqrt{2}\sqrt{6} & \frac{1}{6}\sqrt{3}\sqrt{2} \end{bmatrix}, \text{"*"}, \begin{bmatrix} 3\sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{"*"},$$

$$\begin{bmatrix} \frac{1}{3}\sqrt{3} & \frac{1}{3}\sqrt{3} & \frac{1}{3}\sqrt{3} \\ \frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} \\ \frac{1}{6}\sqrt{6} & -\frac{1}{3}\sqrt{6} & \frac{1}{6}\sqrt{6} \end{bmatrix}$$

```
> print("A alt inverz");
  print("E=",evalm(E),"E Lambda inv=",evalm(E&*Lambdainv));
  Ainv:=evalm(E&*Lambdainv&*transpose(F));
```

"A alt inverz"

$$\text{"E="}, \begin{bmatrix} \frac{1}{3}\sqrt{3} & \frac{1}{2}\sqrt{2} & \frac{1}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{3} & 0 & -\frac{1}{3}\sqrt{6} \\ \frac{1}{3}\sqrt{3} & -\frac{1}{2}\sqrt{2} & \frac{1}{6}\sqrt{6} \end{bmatrix}, \text{"E Lambda inv="}, \begin{bmatrix} \frac{1}{54}\sqrt{3}\sqrt{6} & \frac{1}{12}\sqrt{2}\sqrt{6} & 0 \\ \frac{1}{54}\sqrt{3}\sqrt{6} & 0 & 0 \\ \frac{1}{54}\sqrt{3}\sqrt{6} & -\frac{1}{12}\sqrt{2}\sqrt{6} & 0 \end{bmatrix}$$

$$Ainv := \begin{bmatrix} \frac{-2}{9} & \frac{-1}{6} & \frac{-1}{9} \\ \frac{-1}{18} & 0 & \frac{1}{18} \\ \frac{1}{9} & \frac{1}{6} & \frac{2}{9} \end{bmatrix}$$

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