```
> restart:
  with(linalg):
  print("FOTENGELY TRF");

  a:=1;
  b:=0;
  c:=1;

  A:=matrix(3,3,[0,a,b, a,0,c, b,c,0]);

  P:=charpoly(A,lambda);
  solve(P=0,lambda);

  J:=matrix(3,3,[1,0,0, 0,1,0, 0,0,1]);

  x:=matrix(3,1):
  N:=matrix(3,1,[0,0,0]):



  t:=0;
  print(t,"egy sajatvektora");
  evalm((A-t*J)&*x =N);

  x1:=matrix(3,1,[-z,0,z]);

  print("1-re normalva az elso oszlopa Q-nak");

  q1:=matrix(3,1,[1/sqrt(2),0,-1/sqrt(2)]);


  t:=sqrt(2);
  print(t,"egy sajatvektora");
  evalm((A-t*J)&*x =N);

  x2:=matrix(3,1);
  x2[2,1]=sqrt(2)*x2[1,1];
  x2[3,1]=sqrt(2)*x2[2,1]-x2[1,1];

  x3:=matrix(3,1,[z,sqrt(2)*z,z]);

  print("1-re normalva a masodik oszlopa Q-nak");

  q2:=matrix(3,1,[1/2,sqrt(2)/2,1/2]);
```

```
t:=-sqrt(2);
print(t,"egy sajatvektora");
evalm((A-t*J)&*x =N);

x3:=matrix(3,1);
x3[2,1]=-sqrt(2)*x2[1,1];
x2[3,1]=-sqrt(2)*x2[2,1]-x2[1,1];

x3:=matrix(3,1,[z,-sqrt(2)*z,z]);

print("1-re normalva a harmadik oszlopa Q-nak");
z:=1/2;

q3:=evalm(matrix(3,1,[z,-sqrt(2)*z,z]));
```

```
Warning, new definition for norm
Warning, new definition for trace
```

"FOTENGELY TRF"

$$a := 1$$

$$b := 0$$

$$c := 1$$

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P := \lambda^3 - 2\lambda$$

$$0, \sqrt{2}, -\sqrt{2}$$

$$J := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$t := 0$$

0, "egy sajatvektora"

$$\begin{bmatrix} x_{2,1} \\ x_{1,1} + x_{3,1} \\ x_{2,1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x1 := \begin{bmatrix} -z \\ 0 \\ z \end{bmatrix}$$

"1-re normalva az elso oszlopa Q-nak"

$$q1 := \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ -\frac{1}{2}\sqrt{2} \end{bmatrix}$$

$$t := \sqrt{2}$$

$$\sqrt{2}, \text{"egy sajatvektora"}$$

$$\begin{bmatrix} -\sqrt{2}\, x_{1,1} + x_{2,1} \\ x_{1,1} - \sqrt{2}\, x_{2,1} + x_{3,1} \\ x_{2,1} - \sqrt{2}\, x_{3,1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x2 := \text{array}(1 .. 3, 1 .. 1, [\ ])$$

$$x2_{2,1} = \sqrt{2}\, x2_{1,1}$$

$$x2_{3,1} = \sqrt{2}\, x2_{2,1} - x2_{1,1}$$

$$x3 := \begin{bmatrix} z \\ \sqrt{2}\, z \\ z \end{bmatrix}$$

"1-re normalva a masodik oszlopa Q-nak"

$$q2 := \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2}\sqrt{2} \\ \frac{1}{2} \end{bmatrix}$$

$$t := -\sqrt{2}$$

$$-\sqrt{2}, \text{"egy sajatvektora"}$$

$$\begin{bmatrix} \sqrt{2}\, x_{1,1} + x_{2,1} \\ x_{1,1} + \sqrt{2}\, x_{2,1} + x_{3,1} \\ x_{2,1} + \sqrt{2}\, x_{3,1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x3 := \text{array}(1 .. 3, 1 .. 1, [\ ])$$

$$x3_{2,1} = -\sqrt{2}\, x2_{1,1}$$

$$x2_{3,1} = -\sqrt{2}\, x2_{2,1} - x2_{1,1}$$

$$x3 := \begin{bmatrix} z \\ -\sqrt{2}\, z \\ z \end{bmatrix}$$

"1-re normalva a harmadik oszlopa Q-nak"

$$z := \frac{1}{2}$$

$$q3 := \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2}\sqrt{2} \\ \frac{1}{2} \end{bmatrix}$$

```
> restart:
  with(linalg):
  print("FOTENGELY TRF elokeszites");

  Z:=matrix(3,3,[0,1,2, -1,0,1, -2,-1,0]);
  J:=matrix(3,3,[1,0,0,  0,1,0,  0,0,1]);

  Q:=evalm((Z-J)&*inverse(Z+J));
  QT:=transpose(Q);
  evalm(QT&*Q);

  Lambda:=diag(3,1,-2);

  A:=evalm(Q&*Lambda&*QT);
>
Warning, new definition for norm
Warning, new definition for trace
```

"FOTENGELY TRF elokeszites"

$$Z := \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$J := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q := \begin{bmatrix} \frac{3}{7} & \frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{-3}{7} & \frac{6}{7} \\ \frac{-6}{7} & \frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

$$QT := \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{-6}{7} \\ \frac{6}{7} & \frac{-3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & \frac{3}{7} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Lambda := \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$A := \begin{bmatrix} \frac{55}{49} & \frac{-24}{49} & \frac{-54}{49} \\ \frac{-24}{49} & \frac{-51}{49} & \frac{-78}{49} \\ \frac{-54}{49} & \frac{-78}{49} & \frac{94}{49} \end{bmatrix}$$

```
> print("Az A matrix FOTENGELY TRF-ja");

P:=charpoly(A,lambda);
P:=factor(P);
eigenvectors(A);
lambda:=vector(3,[3,1,-2]);

x1:=matrix(3,1,[3/2, 1, -3]);
x2:=matrix(3,1,[3, -3/2, 1]);
x3:=matrix(3,1,[1, 3, 3/2]);
print("E oszlopai");

x1norm:=sqrt(x1[1,1]^2+x1[2,1]^2+x1[3,1]^2);
e1:=matrix(3,1,[x1[1,1]/x1norm,x1[2,1]/x1norm,x1[3,1]/x1norm]);

x2norm:=sqrt(x2[1,1]^2+x2[2,1]^2+x2[3,1]^2);
e2:=matrix(3,1,[x2[1,1]/x1norm,x2[2,1]/x1norm,x2[3,1]/x1norm]);

x3norm:=sqrt(x3[1,1]^2+x3[2,1]^2+x3[3,1]^2);
e3:=matrix(3,1,[x3[1,1]/x3norm,x3[2,1]/x3norm,x3[3,1]/x3norm]);

E:=matrix(3,3,[
x1[1,1],x2[1,1],x3[1,1],
x1[2,1],x2[2,1],x3[2,1],
x1[3,1],x2[3,1],x3[3,1]]):
print(evalm(A)=evalm(E)*diag(0,sqrt(2),-sqrt(2))*evalm(transpose
(E)));
```

"Az A matrix FOTENGELY TRF-ja"

$$P := \lambda^3 - 2\lambda^2 - 5\lambda + 6$$

$$P := (\lambda - 1)(\lambda - 3)(\lambda + 2)$$

$$\left[3, 1, \left\{\left[\frac{3}{2}, 1, -3\right]\right\}\right], \left[-2, 1, \left\{\left[\frac{2}{3}, 2, 1\right]\right\}\right], \left[1, 1, \left\{\left[3, \frac{-3}{2}, 1\right]\right\}\right]$$

$$\lambda := [3, 1, -2]$$

$$x1 := \begin{bmatrix} \frac{3}{2} \\ 1 \\ -3 \end{bmatrix}$$

$$x2 := \begin{bmatrix} 3 \\ \frac{-3}{2} \\ 1 \end{bmatrix}$$

$$x3 := \begin{bmatrix} 1 \\ 3 \\ \frac{3}{2} \end{bmatrix}$$

"E oszlopai"

$$x1norm := \frac{7}{2}$$

$$e1 := \begin{bmatrix} \frac{3}{7} \\ \frac{2}{7} \\ \frac{-6}{7} \end{bmatrix}$$

$$x2norm := \frac{7}{2}$$

$$e2 := \begin{bmatrix} \frac{6}{7} \\ \frac{-3}{7} \\ \frac{2}{7} \end{bmatrix}$$

$$x3norm := \frac{7}{2}$$

$$e3 := \begin{bmatrix} \frac{2}{7} \\ \frac{6}{7} \\ \frac{3}{7} \end{bmatrix}$$

$$\begin{bmatrix} \frac{55}{49} & \frac{-24}{49} & \frac{-54}{49} \\ \frac{-24}{49} & \frac{-51}{49} & \frac{-78}{49} \\ \frac{-54}{49} & \frac{-78}{49} & \frac{94}{49} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 3 & 1 \\ 1 & \frac{-3}{2} & 3 \\ -3 & 1 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{3}{2} & 1 & -3 \\ 3 & \frac{-3}{2} & 1 \\ 1 & 3 & \frac{3}{2} \end{bmatrix}$$

>