① $\{d[(e^x \sin y)\, dx]\} \wedge \{d[\log(yz)]\} =$

$= \{[d(e^x \sin y)] \wedge dx\} \wedge$

$\wedge \left\{\left[\frac{\partial}{\partial x} \log(yz)\right] dx + \left[\frac{\partial}{\partial y} \log(yz)\right] dy + \left[\frac{\partial}{\partial z} \log(yz)\right] dz\right\} =$

$= \left\{\left[\frac{\partial}{\partial x} e^x \sin y\right] \underbrace{dx \wedge dx}_{0} + \left[\frac{\partial}{\partial y} e^x \sin y\right] dy \wedge dx + \underbrace{\left[\frac{\partial}{\partial z} e^x \sin y\right]}_{0} dz \wedge dx\right\}$

$\wedge \left\{0 \cdot dx + \frac{1}{y} dy + \frac{1}{z} dz\right\} =$

$= \{0 + e^x \cos y\, dy \wedge dx + 0\} \wedge \left\{\frac{1}{y} dy + \frac{1}{z} dz\right\} =$

$= (e^x \cos y) \frac{1}{y}\, \underbrace{dy \wedge dx \wedge dy}_{0} + (e^x \cos y) \frac{1}{z}\, dx \wedge dy \wedge dz =$

$= 0 + \frac{e^x \cos y}{z}\, dx \wedge dy \wedge dz$

② $S(t_1,t_2)=[t_1^2,t_2^2,t_1^2+t_2^2]$ $(t_1,t_2\in[0,1])$,

$$\Omega = \underbrace{xy\,dx\wedge dy}_{\Omega_1} + \underbrace{xyz\,dy\wedge dz}_{\Omega_2}$$

$$\int_S \Omega = \int_S \Omega_1 + \int_S \Omega_2$$

$$\int_S \Omega_1 = \int_{t_1=0}^{1}\int_{t_2=0}^{1} t_1^2 t_2^2 (dt_1^2)\wedge(dt_2^2) =$$

$x=t_1^2$ $y=t_2^2$ $z=t_1^2+t_2^2$

$$=\int_{t_1=0}^{1}\int_{t_2=0}^{1} t_1^2 t_2^2 (2t_1 dt_1)\wedge(2t_2 dt_2) =$$

$$=\int_{t_1=0}^{1}\int_{t_2=0}^{1} t_1^2 t_2^2 (2t_1)(2t_2)\, dt_1\wedge dt_2 =$$

$$=\int_{t_1=0}^{1}\int_{t_2=0}^{1} 4t_1^3 t_2^3\, dt_2\, dt_1 \quad \left[=\int_{t_2=0}^{1}\int_{t_1=0}^{1} 4t_1^3 t_2^3\, dt_1\, dt_2\right]$$

$$=\int_{t_1=0}^{1} 4t_1^3\cdot\frac{1}{4}t_2^4\Big|_{t_2=0}^{1} dt_1 = \int_{t_1=0}^{1} 4t_1^3\cdot\frac{1}{4}\,dt_1 =$$

$$=\int_{t_1=0}^{1} t_1^3\, dt_1 = \underset{\approx}{\frac{1}{4}}$$

$$\int_S \Omega_2 = \int_S xyz \, dy \wedge dz \underset{\substack{\uparrow \\ x=t_1^2 \;\; y=t_2^2 \;\; z=t_1^2+t_2^2}}{=}$$

$$= \int_{t_1=0}^{1} \int_{t_2=0}^{1} t_1^2 t_2^2 (t_1^2+t_2^2) \, (d t_2^2) \wedge [d(t_1^2+t_2^2)] =$$

$$= \int_{t_1=0}^{1} \int_{t_2=0}^{1} [t_1^4 t_2^2 + t_1^2 t_2^4] \, (2t_2 \, dt_2) \wedge (2t_1 \, dt_1 + 2t_2 \, dt_2) =$$

$$= \int_{t_1=0}^{1} \int_{t_2=0}^{1} [t_1^4 t_2^2 + t_1^2 t_2^4] \, [4 t_1 t_2^2 \, dt_2 \wedge dt_1 + 4 t_2^2 \underbrace{dt_2 \wedge dt_2}_{0}] =$$

$$= \int_{t_1=0}^{1} \int_{t_2=0}^{1} 4 [t_1^5 t_2^4 + t_1^3 t_2^6] \, \underbrace{dt_2 \wedge dt_1}_{\ominus} =$$

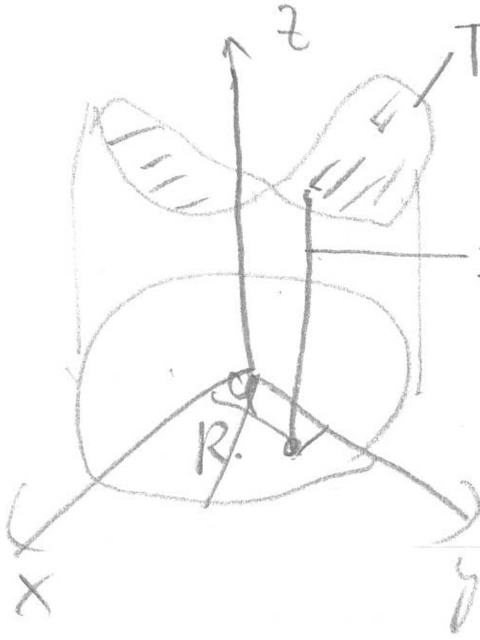
$$= \int_{t_1=0}^{1} \int_{t_2=0}^{1} (-4) (t_1^5 t_2^4 + t_1^3 t_2^6) \, dt_1 \wedge dt_2 =$$

$$= \int_{t_1=0}^{1} \int_{t_2=0}^{1} (-4) (t_1^5 t_2^4 + t_1^3 t_2^6) \, dt_2 \, dt_1 =$$

$$= (-4) \left(\frac{1}{6} \cdot \frac{1}{5} + \frac{1}{4} \cdot \frac{1}{7} \right) = -\frac{2}{15} - \frac{1}{7}$$

$$\int_S \Omega = \frac{1}{4} - \frac{2}{15} - \frac{1}{7} = \frac{105-28-60}{420} = 17/420$$

③



TETŐ (STADIONÉ – FORGÁSI HIPERBOLOID)

$$z = \frac{h_1+h_2}{2} + (h_2-h_1)\frac{xy}{R^2}$$

$R = 50m \quad h_1 = 25m \quad h_2 = 15m$

TERÜLET = ?

$$\text{TERÜLET} = \int_{\text{PARTAR}} \text{Jacdet} = \int_Q \sqrt{(dx\wedge dy)^2 + (dy\wedge dz)^2 + (dz\wedge dx)^2}$$

PARAMÉTEREZÉS

$$x = r\cos\varphi \quad y = R\sin\varphi \quad z = \frac{h_1+h_2}{2} + (h_2-h_1)\frac{r^2}{R^2}\cos\varphi\sin\varphi$$

PARAMÉTERTART: $R\in[0,50)$ $\varphi\in[0,2\pi]$

$$Q : [0,50)\times[0,2\pi] \longrightarrow \mathbb{R}^3$$

$$(r,\varphi) \mapsto \begin{array}{l} \begin{bmatrix} r\cos\varphi \\ r\sin\varphi \\ \frac{h_1+h_2}{2} + (h_2-h_1)\frac{r^2}{R^2}\underbrace{\sin\varphi\cos\varphi}_{\frac{1}{2}\sin 2\varphi} \end{bmatrix} \end{array} \begin{array}{l} Q_1 \\ Q_2 \\ Q_3 \end{array}$$

$$\text{TERÜLET} = \int_{r=0}^{50} \int_{\varphi=0}^{2\pi} \sqrt{\det(Q'^T \cdot Q')}\, d\varphi\, dr$$

$$Q' = Q'(r,\varphi) = \begin{bmatrix} \partial Q_1/\partial r & \partial Q_1/\partial \varphi \\ \partial Q_2/\partial r & \partial Q_2/\partial \varphi \\ \partial Q_3/\partial r & \partial Q_3/\partial \varphi \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \\ \underbrace{(h_2-h_1)}_{H}\frac{r}{R^2}\sin 2\varphi & \underbrace{(h_2-h_1)}_{H}\cos 2\varphi \frac{r^2}{R^2} \end{bmatrix}$$

$$Q'^T Q = \left[\langle Q'_{i.\text{oszl}} \mid Q'_{j.\text{oszl}} \rangle\right]_{i,j=1}^{2} =$$

$$= \begin{bmatrix} \underbrace{\cos^2\varphi + \sin^2\varphi}_{1} + \frac{H^2 r^2}{R^4}\sin^2 2\varphi & \text{UAZ} \\ \frac{H^2 r^3}{R^4}\sin 2\varphi \cos 2\varphi & r^2 + \frac{H^2 r^4}{R^4}\cos^2 2\varphi \end{bmatrix} =$$

$$\det(Q'^T Q) = \frac{r^2 (R^4 + H^2 r^2 \cos^2 2\varphi + H^2 r^2 \sin^2 2\varphi)}{R^4}$$

$$\text{Jac}\,Q| = \sqrt{} = r\sqrt{1 + \frac{H^2 r^2}{R^4}}$$

$$\text{TERÜLET} = \int_{r=0}^{R} \int_{\varphi=0}^{2\pi} \sqrt{1 + (H^2/R^4)\, r^2} \cdot r \, d\varphi\, dr \underset{s=r^2}{=}$$

$$= 2\pi \int_{s=0}^{R^2} \sqrt{1 + (H^2/R^4)\, s}\, ds = \frac{\left(\sqrt{R^2+H^2}^{\,3} - R^3\right) R}{H^2}$$

Elliptikus $\int$

$$Q : [0, 2\pi] \ni \varphi \mapsto \begin{bmatrix} A\cos\varphi \\ B\sin\varphi \end{bmatrix}$$

$$\left[Q[0,2\pi] \text{ hossza}\right] = \int_Q \mathrm{Jac}\, Q \, d\varphi =$$

$$= \int_{\varphi=0}^{2\pi} \sqrt{\det(Q'^T Q')}\, d\varphi$$

$$Q' = \begin{bmatrix} -A\sin\varphi \\ B\cos\varphi \end{bmatrix} \qquad Q'^T Q' = A^2\sin^2\varphi + B^2\cos^2\varphi$$

$$\text{KERÜLET} = \int_{\varphi=0}^{2\pi} \sqrt{A^2\sin^2\varphi + B^2\cos^2\varphi}\, d\varphi$$

NINCS VÉGES ELEMI KÉPLET

$\longrightarrow$ új speciális fgv-ek

ELLIPTIKUS FGV-EK

$$\text{TERÜLET} = \int_{r=0}^{50} 2 \int_{\psi=0}^{4\pi} \sqrt{r^2 + 100\cos^2\psi} \cdot \frac{1}{2}\, d\psi =$$

$\psi = 2\varphi \quad d\psi = 2\, d\varphi$

$$= \int_{r=0}^{50} 2 \cdot \mathrm{Ellper}\left(\sqrt{r^2+100}, 10\right) d\psi$$