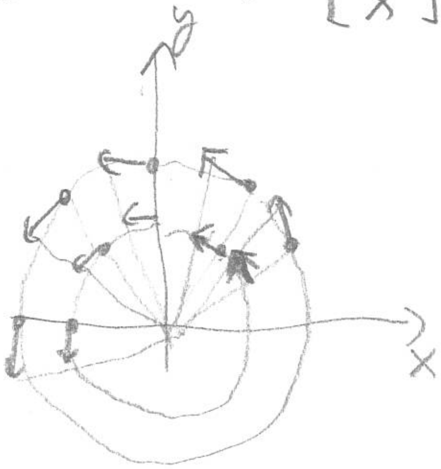
KLASSZIKUS VEKTORANALÍZIS

Vektormező $\mathbb{R}^N$-en

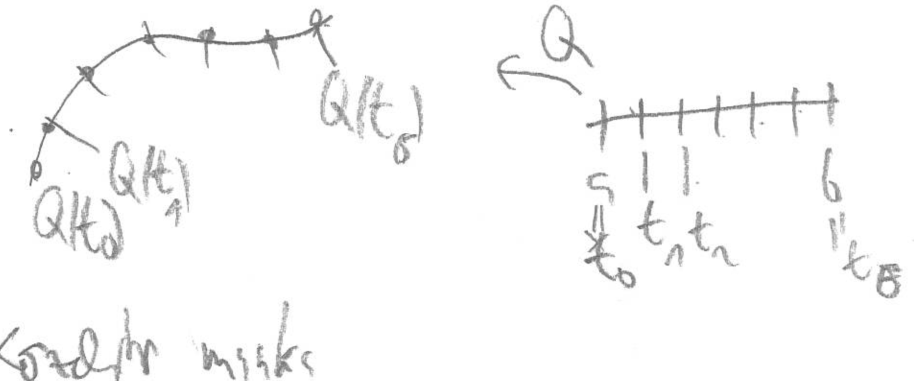
$F : \mathbb{R}^N \to \mathbb{R}^N$

Pl. $N=2$ $\quad F = \begin{bmatrix} -y \\ x \end{bmatrix} \cdot \frac{1}{3}$



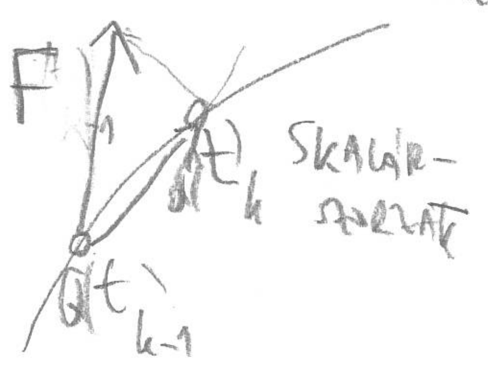
Vektormező munkája görbén

$Q : [a,b] \to \mathbb{R}^N \qquad F : \mathbb{R}^N \to \mathbb{R}^N$



Közelítő munka

$$\tilde{W} = \sum_{k=1}^{n} \tilde{W}_k = \sum_{k=1}^{n} \langle F(Q(t_k)) \mid Q(t_k) - Q(t_{k-1}) \rangle =$$



SKALÁR-SZORZAT

x_1-KOORD

$$= \sum_{k=1}^{n} \Big\{ F_1(Q(t_{k-1})) \, [Q_1(t_k) - Q_1(t_{k-1})] + \dots + F_N(Q(t_{k-1})) \, [Q_N(t_k) - Q_N(t_{k-1})]$$

$$Q_j(t_k) - Q_j(t_{k-1}) \approx \left.\frac{\partial Q_j}{\partial t}\right|_{t=t_{k-1}} \cdot (t_k - t_{k-1}) \qquad j=1,\dots,N$$

$$W \approx \sum_{k=1}^{N} \left\{ F_1(Q(t_{k-1})) \cdot \left.\frac{\partial Q_1}{\partial t}\right|_{t=t_{k-1}} \cdot (t_k - t_{k-1}) + \dots + \right.$$

$$\left. + F_N(Q(t_{k-1})) \left.\frac{\partial Q_N}{\partial t}\right|_{t=t_{k-1}} \underbrace{(t_k - t_{k-1})}_{\text{[illegible]}} \right\}$$

BEOSZTÁS MAX $\searrow 0$

$$W = \int_{t=a}^{b} \left\{ F_1(Q(t)) \cdot \frac{\partial Q_1}{\partial t} + \dots + F_N(Q(t)) \frac{\partial Q_N}{\partial t} \right\} dt$$

CÉLSZERŰBB JELÖLÉS

$$Q_1 = [Q \ x_1 \ \text{KOORD}] \longrightarrow x_1 \qquad Q_1(t) = x_1(t)$$

$$Q_2 = [Q \ x_2 \ \text{KOORD}] \longrightarrow x_2 \qquad Q_2(t) = x_2(t)$$

$$W = \int_a^b \left\{ F_1(x_1(t), \dots, x_N(t)) \cdot \frac{\partial x_1}{\partial t} dt + \dots + \right.$$

$$\left. + F_N(x_1(t), \dots, x_N(t)) \cdot \underbrace{\frac{\partial x_N}{\partial t} dt}_{dx_N} \right\}$$

$$\boxed{W = \int_Q \sum_{j=1}^{N} F_j(x_1, \dots, x_N) \, dx_j}$$

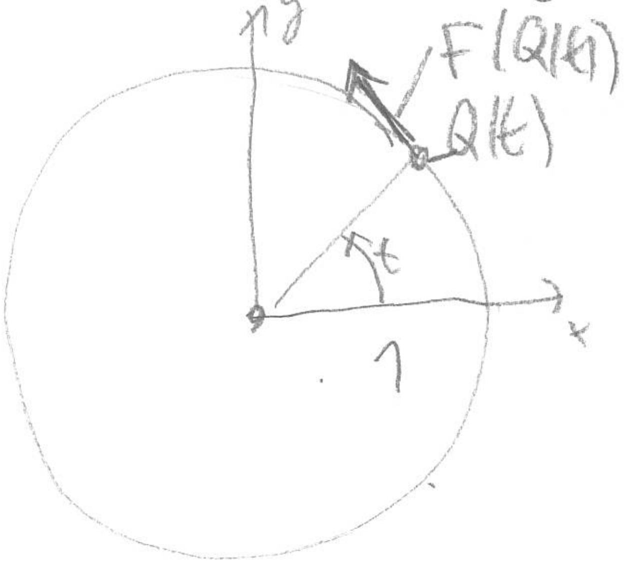
Példa [ÖRVÉNY]

$$F:\mathbb{R}^2\to\mathbb{R}^2 \quad p\mapsto\begin{bmatrix}-y(p)\\x(p)\end{bmatrix}\cdot\frac{1}{3} \quad F=\begin{bmatrix}-y\\x\end{bmatrix}\cdot\frac{1}{3}$$

F_1

F_2

$$Q:[0,2\pi]\to\mathbb{R}^2 \quad t\mapsto\begin{bmatrix}\cos t\\ \sin t\end{bmatrix}$$



TELJES KÖR

$x=\cos t$ Az $\begin{matrix}x\\ \vdots\\ y\end{matrix}$ KOORD. ÉRTÉKE Q-n a t IDŐBEN

$y=\sin t$

$$W=\int_Q\left[F_1\,dx+F_2\,dy\right]=\int_Q\left[-y\,dx+x\,dy\right]=$$

$$=\int_{t=0}^{2\pi}\left\{-\sin t\,\underbrace{d\cos t}_{(\cos t)'\,dt}+\cos t\cdot\underbrace{d\sin t}_{(\sin t)'\,dt}\right\}=$$

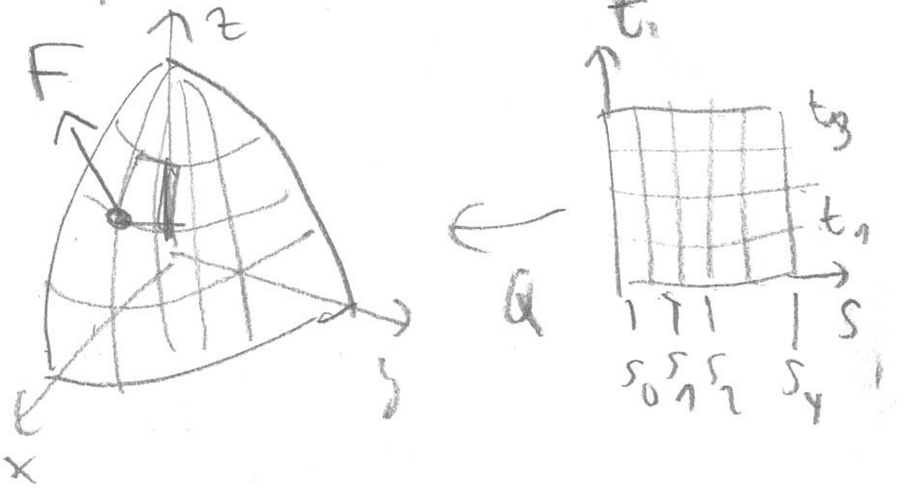
$$=\int_{t=0}^{2\pi}\left\{-\sin t\,(-\sin t)\,dt+\cos t\cdot\cos t\cdot dt\right\}=$$

$$=\int_{t=0}^{2\pi}\underbrace{(\sin^2 t+\cos^2 t)}_{1}\,dt=2\pi$$

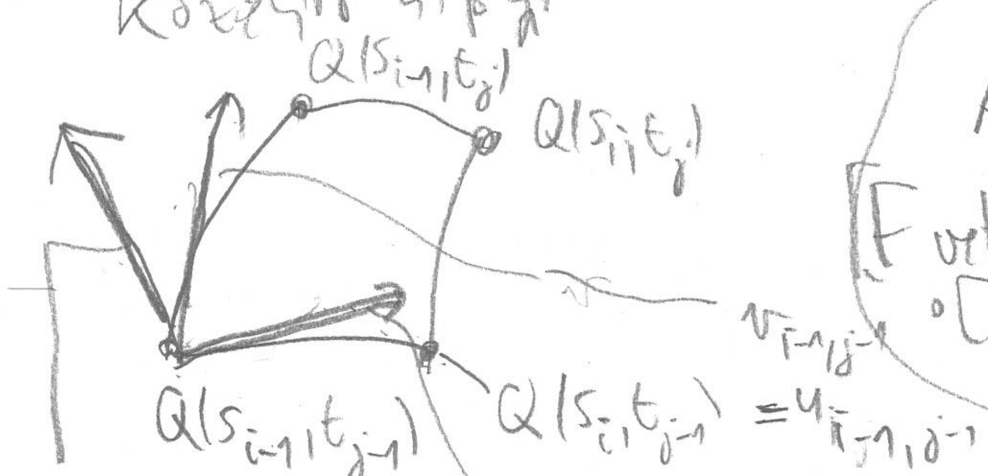
Vektormező ÁTFOLYÁSA felületen FLUXUS

$F : \mathbb{R}^3 \to \mathbb{R}^3$ vektormező $\mathbb{R}^3$-ban

$Q : [a_1, b_1] \times [a_2, b_2] \to \mathbb{R}^3$ 2-dim felület db



közelítő átfolyás



ÁTFOLYÁS $\approx$

[F vetülete u,v NORMÁLISRA] · [u,v PAR TER]

$F(Q(s_{i-1}, t_{j-1}))$

$$\left.\frac{\partial Q}{\partial s}\right|_{(s,t)=(s_{i-1},t_{j-1})} \cdot (s_i - s_{i-1})$$

$$\Phi = \sum_{i=1}^{4} \sum_{j=1}^{3} \phi_{i,j}$$

$$\phi_{i,j} \approx F_1 \cdot (u_2 v_3 - v_2 u_3) - F_2 (u_1 v_3 - v_1 u_3) + F_3 (u_1 v_2 - v_1 u_2)$$

$\det(F, u, v)$ $\quad u = u_{i-1,j-1} \quad v = v_{i-1,j-1}$

$$\det(F, u, v) = \det\begin{bmatrix} F_1 & u_1 & v_1 \\ F_2 & u_2 & v_2 \\ F_3 & u_3 & v_3 \end{bmatrix} = F_1$$

$$u_{i-1,j-1} = \left.\frac{\partial Q}{\partial s}\right|_{\substack{(s,t)= \\ (s_{i-1}, t_{j-1})}} \cdot (s_i - s_{i-1})$$

$$v_{i-1,j-1} = \left.\frac{\partial Q}{\partial t}\right|_{\substack{(s,t)= \\ (s_{i-1}, t_{j-1})}} \cdot (t_j - t_{j-1})$$

BEOSZTÁS MAX $\searrow 0$

$$\Phi = \int_{s=a_1}^{b_1} \int_{t=a_2}^{b_2} \Big\{ F_1(Q(s,t)) \det\begin{bmatrix} \partial Q_2/\partial s & \partial Q_2/\partial t \\ \partial Q_3/\partial s & \partial Q_3/\partial t \end{bmatrix} +$$

$$+ F_2(Q(s,t)) \det\begin{bmatrix} \partial Q_3/\partial s & \partial Q_3/\partial t \\ \partial Q_1/\partial s & \partial Q_1/\partial t \end{bmatrix} +$$

$$+ F_3(Q(s,t)) \det\begin{bmatrix} \partial Q_1/\partial s & \partial Q_1/\partial t \\ \partial Q_2/\partial s & \partial Q_2/\partial t \end{bmatrix} \Big\} \, dt\, ds$$

ÚJ ALGEBRAI TECHNIKA KELL

A DETERMINÁNSOK MIATT