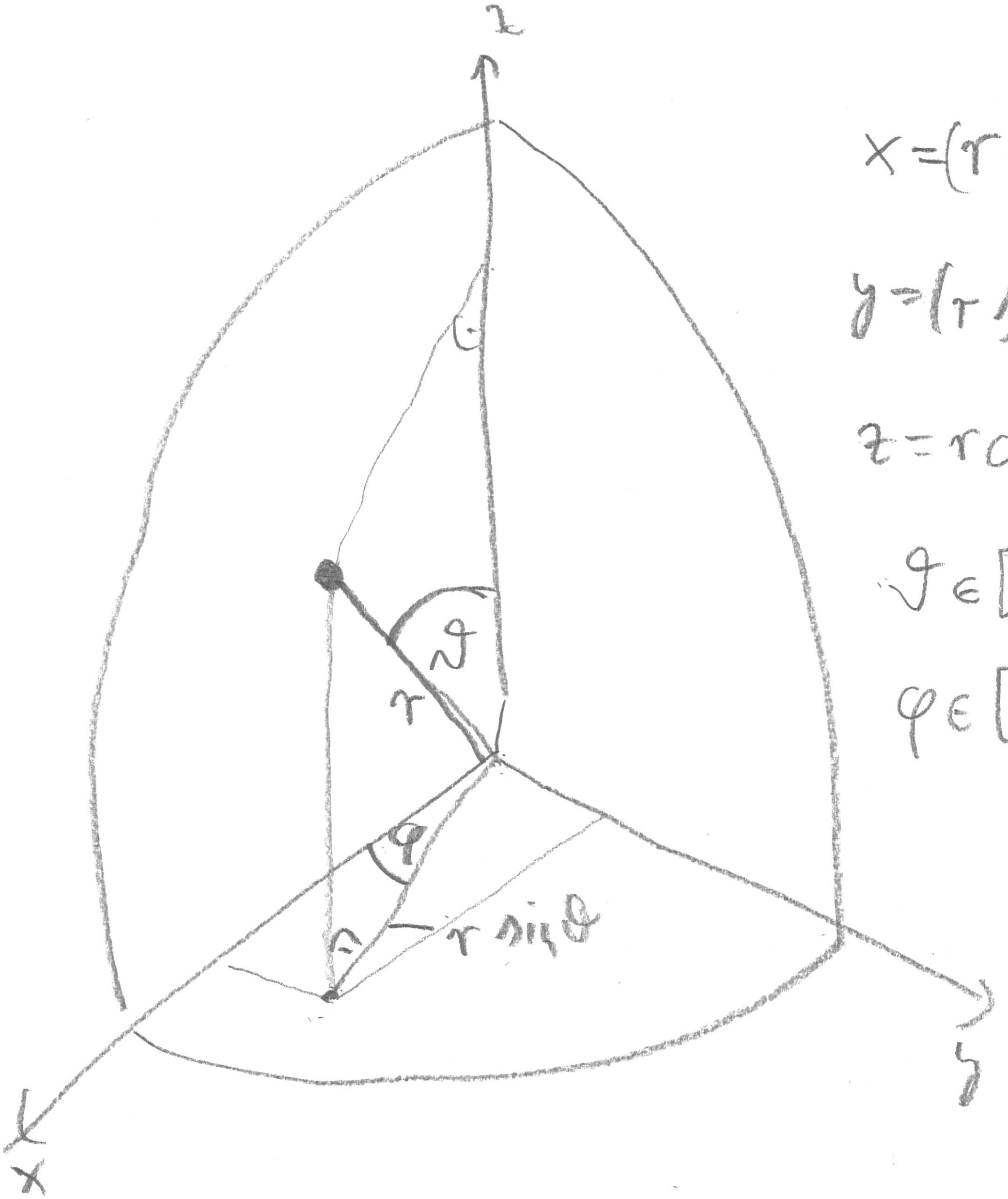
PARAMÉTEREZETT FELÜLET

Pl 8-ad GÖMB

r sugarú



$x = (r \sin\vartheta)\cos\varphi$

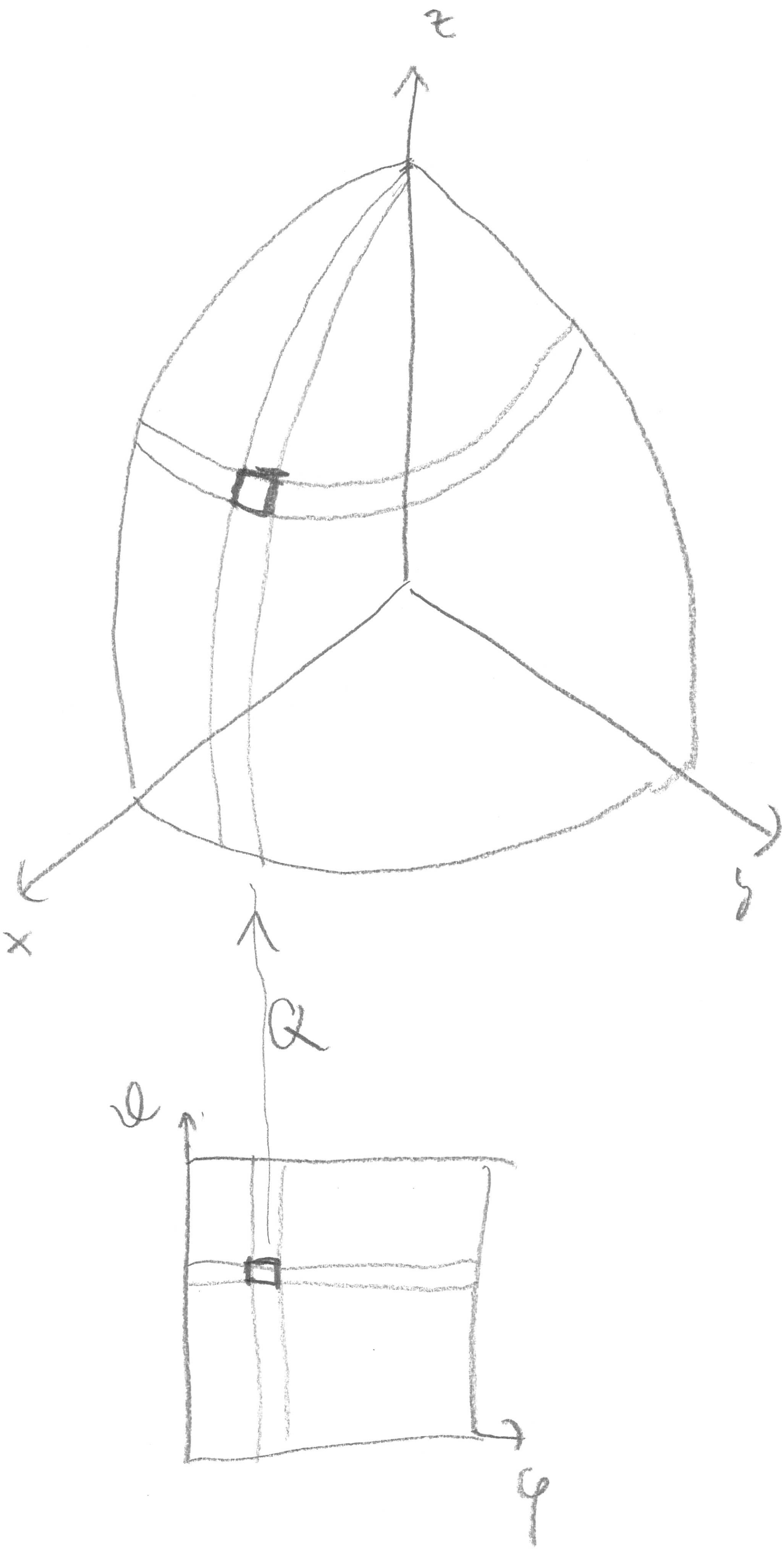
$y = (r \sin\vartheta)\sin\varphi$

$z = r\cos\vartheta$

$\vartheta \in [0, \pi/2]$

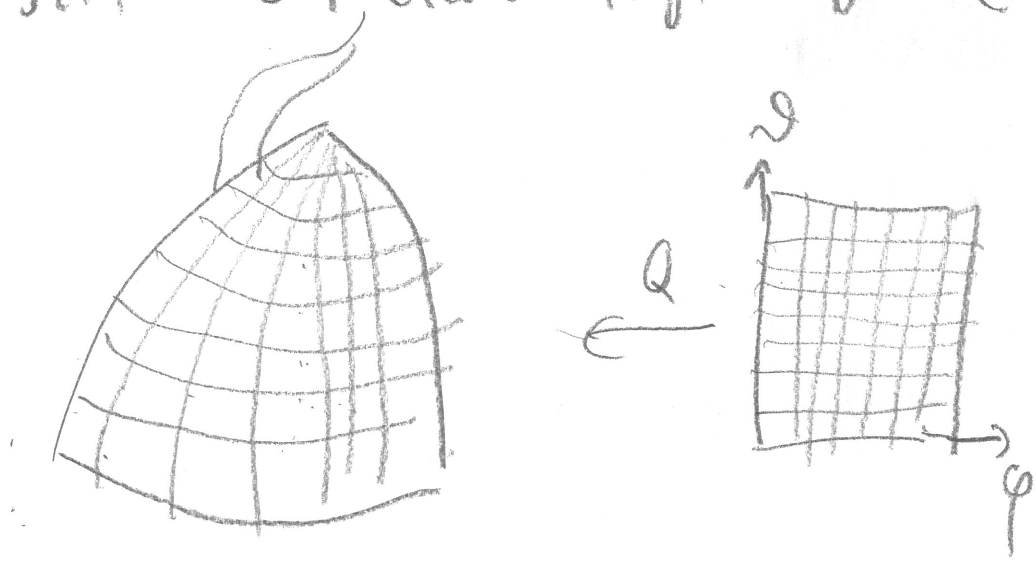
$\varphi \in [0, \pi/2]$

KÖZELÍTŐ PARALLELOGRAMMÁK

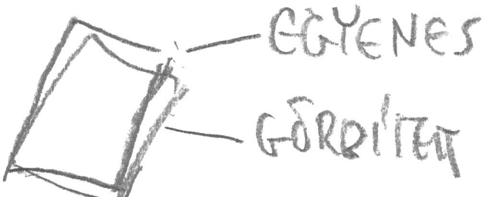


GÖRBÍTETT
PARALLELOGR.

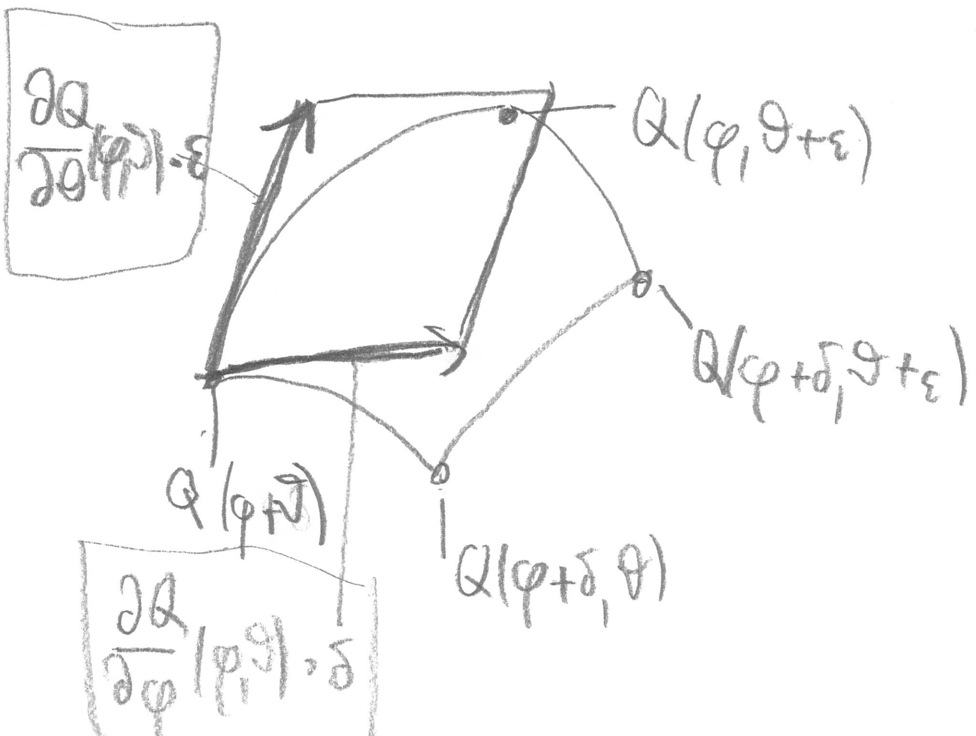
Felszín = Σ {lefedő diszjunkt görbe paral.}



ELVÁRÁS: diam(GÖRBE PAR) ↘ 0 ⇒



$$\frac{\text{Felszín(GÖRBE PAR)}}{\text{Felszín(EGYENES PAR)}} \to 1$$

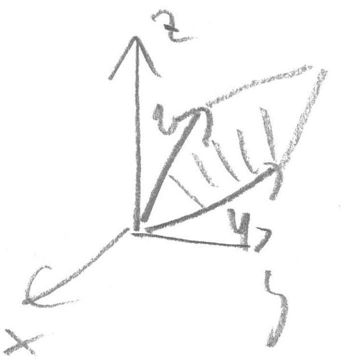


FELSZÍN FORMULA

LIM $\varepsilon, \delta \to 0 \implies$

$$\text{Felszín} = \int_{\vartheta=0}^{\pi/2} \int_{\varphi=0}^{\pi/2} \left\langle \underbrace{\frac{\partial Q}{\partial \varphi}}_{\text{VEKTOR}}, \frac{\partial Q}{\partial \vartheta} \text{ OLD PAR TER} \right\rangle d\varphi\, d\vartheta$$

Kérdés: $\langle u, v \text{ OLD PAR TER} \rangle = ?$



$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$L := \begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \\ u_3 & v_3 \end{bmatrix}$$ egy leképezés $\mathbb{R}^2 \to \mathbb{R}^3$

$$\langle u, v \text{ OLD PAR} \rangle = L\,[0,1]^2 = L\,\square$$

SVD $\quad L = \underbrace{R_1}_{\substack{\text{ORT} \\ 3\times 3}} \Lambda \underbrace{R_2^T}_{\substack{\text{ORT} \\ 2\times 2}} \qquad \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \\ 0 & 0 \end{bmatrix}$

(*) L (alakzat) felszín nem változik EGYBEVÁGÓSÁGI TRANSZF-RA és DESCARTES KOORD-ban SZÁMÍTVA

$$\text{Felszín}(L\,\square) = \text{Felszín}(R_1 \Lambda R_2\,\square) = \text{Felszín}(\Lambda\,\square) =$$

$$= \underline{\lambda_1 \lambda_2} \quad !!!$$

$\lambda_1 \lambda_2$ KISZÁMÍTÁSA

$$L = R_1 \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} R_2^T \qquad L^T L = R_2 \underbrace{\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}}_{\Lambda^T} \underbrace{\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}}_{\Lambda} R_2^T =$$

$$= R_2 \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} R_2^T$$

$$\lambda_1^2 \lambda_2^2 = \det \begin{pmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{pmatrix} = \det R_2 \begin{bmatrix} \lambda_1^2 & \\ & \lambda_2^2 \end{bmatrix} R_2^T =$$

$$= \det(L^T L)$$

$$\lambda_1 \lambda_2 = \sqrt{\det(L^T L)}$$

JACOBI DETERMINANS

$$Q' = \begin{bmatrix} \underbrace{\frac{\partial Q}{\partial \varphi}}_{\text{vekt}} & \underbrace{\frac{\partial Q}{\partial \vartheta}}_{\text{vekt } \mathbb{R}^3\text{-ből}} \end{bmatrix}$$

Q Jacobi mátrixa $\approx L$

$$\text{Jac}\,Q = \left[\det(Q'^T Q')\right]$$

$$\boxed{\begin{aligned} &\text{felszín}\,(Q([a,b] \times [c,d])) = \\ &\int_{\varphi=a}^{b} \int_{\vartheta=c}^{d} \text{Jac}(Q)\, d\vartheta\, d\varphi \end{aligned}}$$