A Fourier-trf $\langle\cdot|\cdot\rangle$-tartása $L^2(\mathbb{R})$-en

$$\mathcal{F}(f(t)|\omega) := \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{\infty} f(t)\, e^{-it\omega}\, d\omega \quad , \quad \underline{f \in L^1(\mathbb{R})}$$

INVERZ:

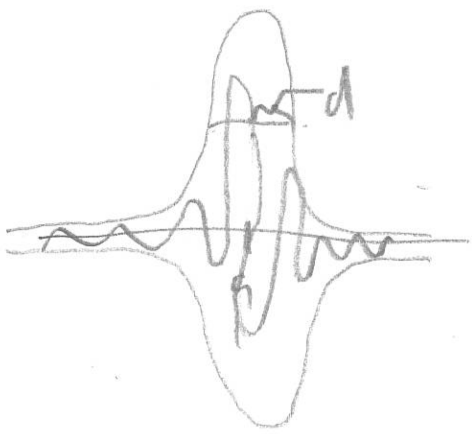
$$\mathcal{F}^{-1}(g(\omega)|t) := \frac{1}{\sqrt{2\pi}} \int_{\omega=-\infty}^{\omega} g(\omega)\, e^{i\omega t}\, d\omega \qquad \underline{g \in L^1(\mathbb{R})}$$

Jel felbontása $\underline{\sin \omega t, \cos \omega t}$ alaphangokra (kompl. alak)

GÁBOR Dénes VÉGES ENERGIÁJÚ ALAPHANGJAI:

$$\frac{e^{-(t-a)^2/(2d^2)}}{d\sqrt{2\pi}} \cdot \begin{Bmatrix} \sin \omega t \\ \cos \omega t \end{Bmatrix}$$

$c\, e^{i\omega(t-b)}$ kompl. alak



Tudjuk: $\left\{ e^{-(t-a)^2/(2d^2)} : d>0 \right\}$ LIN KOMB TETSZŐLEGESEN MEGKÖZELÍTIK A FOLYT KOMP. TARTÓJÚ FGV-EKET → SŰRŰN VANNAK $L^2(\mathbb{R})$-ben

<u>Tétel</u> $\underline{\psi_k(t) := e^{-(t-a_k)^2/(2d_k^2)}}$ $(k=1,2)$ esetén

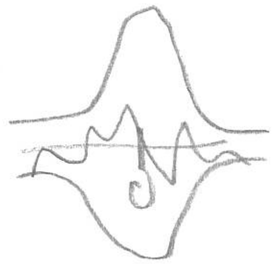
$\varphi_k(\omega) = \mathcal{F}(\psi_k(t)|\omega)$ Fourier-trf-ja

$$\boxed{\langle \psi_1 | \psi_2 \rangle = \langle \varphi_1 | \psi_2 \rangle}$$

bit $\varphi_k(\omega) = \frac{d_k}{\sqrt{2}} e^{-\frac{1}{4}\omega(4ia_k + \omega d_k^2)}$ $(k=1,2)$

MAPLE with(inttrans) $\uparrow$ standard

$\varphi_k \in L^2(\Omega)$



$$\underline{\langle\psi_1|\psi_2\rangle} = \int_{t=-\infty}^{\infty} \psi_1(t)\,\psi_2(t)\,dt =$$

$$= \int_{t=-\infty}^{\infty} e^{-(t-a_1)^2/(2d_1^2) - (t-a_2)^2/(2d_2)^2}\,dt =$$

$$= \underline{\frac{d_1 d_2 \sqrt{\pi}}{\sqrt{d_1^2+d_2^2}} e^{-\frac{(a_1-a_2)^2}{d_1^2+d_2^2}}} \quad \text{(MAPLE)}$$

$$\langle\psi_1|\psi_2\rangle = \int_{\omega=-\infty}^{\infty} \psi_1(\omega)\,\overline{\psi_2(\omega)}\,d\omega =$$

$\uparrow$ konjugiert

$$= \int_{\omega=-\infty}^{\infty} \frac{d_1 d_2}{2} e^{-\frac{1}{4}\omega(4ia_1+\omega d_1^2) - \frac{1}{4}\omega(-4ia_2+\omega d_2^2)}\,d\omega =$$

$$= \frac{d_1 d_2 \sqrt{\pi}}{\sqrt{d_1^2+d_2^2}} e^{-\frac{(a_1-a_2)^2}{d_1^2+d_2^2}} \quad \text{q.e.d.}$$

__Megjegyzés__ $A>0$, $B, C \in \mathbb{C}$ esetén ÁLTALÁBAN

$$\mathcal{F}\left(e^{-A^2t^2+Bt+C}, \omega\right) = \frac{1}{\sqrt{2}A} e^{-\tilde{A}^2\omega^2+\tilde{B}\omega+\tilde{C}}$$

$$\tilde{A} = \frac{1}{2A}, \quad \tilde{B} = -\frac{i}{2}\frac{B}{A^2}, \quad \tilde{C} = \frac{B^2+4A^2C}{4A^2}$$

A Gábor Dénes-féle ALAPHANGRA

$$\mathcal{F}\left(e^{-(t-a)^2/2d^2} \cdot e^{i\nu(\omega-b)}/[d\sqrt{2\pi}], \omega\right) =$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d^2\omega^2+(d^2\nu-ia)\omega-\frac{1}{2}\nu[d^2\nu+2i(b-a)]}$$

[Számítás: MAPLE, Gabor_Denes_fgv.mws]