$$G = \frac{\partial g}{\partial x_1} dx_2 \wedge \dots \wedge dx_N + \frac{\partial g}{\partial x_2} dx_1 \wedge dx_3 \wedge \dots \wedge dx_N + \dots =$$

$$= \sum_{k=1}^{N} \frac{\partial g}{\partial x_i} (-1)^{i-1} dx_1 \wedge \dots \wedge \widehat{dx_k} \wedge \dots \wedge dx_N$$

$$0 = \int_{\partial\Omega} fG = \int_{\Omega} d(fG) = \int_{\Omega} [df \wedge G + f\, dG]$$

$f|_{\partial\Omega} = 0$

$$df \wedge G = \left[\frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \dots + \frac{\partial f}{\partial x_N} dx_N\right] \wedge$$

$$\wedge \left[\frac{\partial g}{\partial x_1} dx_2 \wedge \dots \wedge dx_N + \dots + (-1)^{N-1} \frac{\partial g}{\partial x_N} dx_1 \wedge \dots \wedge dx_{N-1}\right] =$$

$$= \frac{\partial f}{\partial x_1} \frac{\partial g}{\partial x_1} dx_1 \wedge \dots \wedge dx_N + \dots + \frac{\partial f}{\partial x_N} \frac{\partial g}{\partial x_N} dx_1 \wedge \dots \wedge dx_N$$

$$\int_{\Omega} df \wedge G = \int_{\Omega} \sum_{i=1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial g}{\partial x_i} \underbrace{dx_1 \wedge \dots \wedge dx_N}_{dx_1 \dots dx_N \text{ KLASIKU}} = \int \left\langle \frac{\partial f}{\partial x} \middle| \frac{\partial g}{\partial x} \right\rangle dx_1 \dots dx_N$$

$$dG = \left[\frac{\partial}{\partial x_1}\left(\frac{\partial g}{\partial x_1}\right) dx_1 \wedge dx_2 \wedge \dots \wedge dx_N + 0\right] + \dots +$$

$$+ \left[0 + \frac{\partial}{\partial x_N}\left(\frac{\partial g}{\partial x_N}\right) \cdot (-1)^{N-1} \underbrace{dx_N \wedge dx_1 \wedge \dots \wedge dx_{N-1}}_{N-1 \text{ RAZY}}\right] =$$

$$= \sum_{i=1}^{N} \frac{\partial^2 g}{\partial x_i^2} dx_1 \wedge \dots \wedge dx_N = \Delta g \cdot dx_1 \wedge \dots \wedge dx_N$$

$$0 = \int_{\partial\Omega} fG = \int_{\Omega} \left\langle \frac{\partial f}{\partial x} \middle| \frac{\partial g}{\partial x} \right\rangle dx_1 \dots dx_N + \int f \Delta g \, dx_1 \dots dx_N$$

$f \leftrightarrow g$

$$0 = \int_{\partial\Omega} Fg = \int_{\Omega} \left\langle \frac{\partial g}{\partial x} \middle| \frac{\partial f}{\partial x} \right\rangle dx_1 \dots dx_N + \int g \Delta f \, dx_1 \dots dx_N$$

$\left\langle \frac{\partial f}{\partial x} \middle| \frac{\partial g}{\partial x} \right\rangle = \left\langle \frac{\partial g}{\partial x} \middle| \frac{\partial f}{\partial x} \right\rangle \Rightarrow$

$\int_\Omega f \Delta g - \int g \Delta f = 0$ ὅπου $f|_{\partial\Omega} = g|_{\partial\Omega} = 0$

$\Delta f = \lambda f$, $\Delta g = \mu g \Rightarrow \int f(\mu g) = \int g(\lambda f) = 0$, $\mu \int fg - \lambda \int fg = 0$

$(\lambda - \mu)\langle f|g\rangle = 0$, $\langle f|g\rangle = 0$ ὅπου $\lambda \neq \mu$