Peremfeltételek

1) $u(x,t)\equiv 0,\ x\in\partial\Omega$

FIXÁLT HATÁR

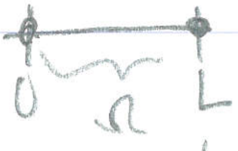
DOBOK (2D)

2) 

$\nabla u \perp \partial u$

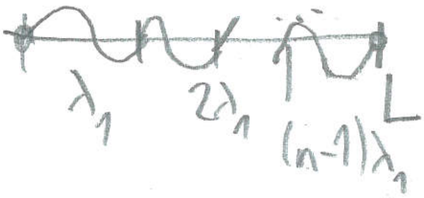
$\nabla u(x) \parallel N_{\partial\Omega}(x),\ x\in\partial\Omega$

SZABAD SZÉLEK

Példák ① 1D: 

$$\boxed{u(t,x)=\sin\left(2\pi\frac{t}{\lambda/c}\right)\cdot\sin\left(2\pi\frac{x}{\lambda_1}\right)}$$

RÖGZ VÉGŰ HÚR

a) 

$L=n\lambda_1$, $\frac{1}{\lambda_0^2}=\frac{1}{\lambda_1^2}$ $\lambda_0=\lambda_1=\frac{L}{n}$

b) 

$L=(n+\frac{1}{2})\lambda_1$ $\lambda_0=\lambda_1=\frac{L}{n+\frac{1}{2}}$

② 

FUVOLA

SZABAD VÉGŰ REZGÉST

ADTATNA A LEVEGŐ MOLEKULÁK

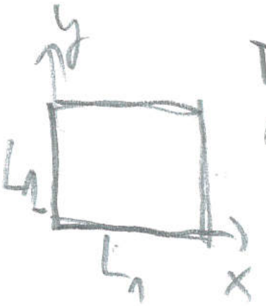
a) 

Itt is a) $\lambda_0=\lambda_1=\frac{L}{n}$

b) 

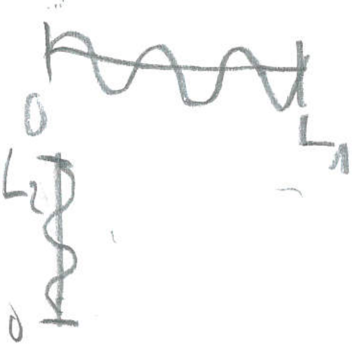
b) $\lambda_0=\lambda_1=\frac{L}{n+\frac{1}{2}}$

③ TÉGLALAP ALAKÚ DOB



$$u(x,t) = \sin\left(2\pi \frac{t}{\lambda_0/c}\right) \sin\left(2\pi \frac{x}{\lambda_1}\right) \sin\left(2\pi \frac{y}{\lambda_2}\right)$$

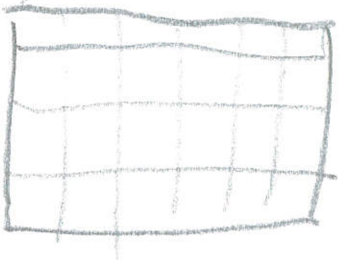
$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$



$\lambda_1 = \frac{L_1}{n}$ VAGY $\lambda_1 = \frac{L_1}{n+\frac{1}{2}}$

$\lambda_2 = \frac{L_2}{m}$ VAGY $\lambda_2 = \frac{L_2}{m+\frac{1}{2}}$

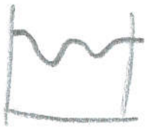
LISSAJOUS-VONALAK



AHOL $u(x,t) \equiv 0 \;\; \forall t$

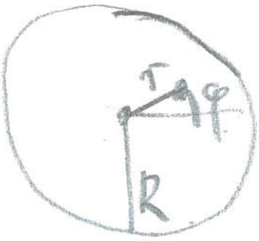
(homokot rászórva elhagyják)

④ HASONLÓ A SZABAD PEREM ESETE

Fizikailag ~  ÚSZÓMEDENCE MAGAS ADLEKKEL

⑤ KÖR ALAKÚ DOB

Szeptcids koordináták t, r, φ



$$u(x,t) = \sin\left(2\pi \frac{t}{\lambda_0/c}\right) \cdot f_1(r)\, f_2(\varphi)$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u = c^2 \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right] f_1(r)\, f_2(\varphi)$$

$$-4\pi^2 \frac{c^2}{\lambda_0^2} \sin\left(2\pi \frac{t}{\lambda_0/c}\right) f_1(r)\, f_2(\varphi) \equiv$$

$$c^2 \sin\left(2\pi \frac{t}{\lambda_0/c}\right) \cdot \left[f_1''(r)\, f_2(\varphi) + \frac{1}{r} f_1'(r)\, f_2(\varphi) + \frac{1}{r^2} f_1(r)\, f_2''(\varphi)\right]$$

$$-\frac{4\pi^2}{\lambda_0^2} = \frac{f_1''(r)}{f_1(r)} + \frac{1}{r}\frac{f_1'(r)}{f_1(r)} + \frac{1}{r^2}\frac{f_2''(\varphi)}{f_2(\varphi)} \qquad / \cdot r^2$$

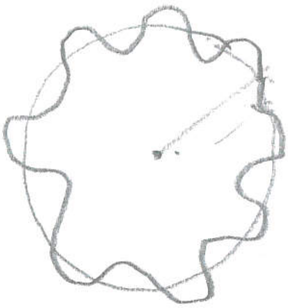
↑ const

$$\frac{f_2''(\varphi)}{f_2(\varphi)} = -\left[\frac{4\pi^2}{\lambda_0^2} r^2 + \frac{f_1''(r)}{f_1(r)} \cdot r^2 + \frac{f_1'(r)}{f_1(r)} \cdot r\right]$$

csak φ-től — csak r-től függ

$$\frac{f''(\varphi)}{f_2(\varphi)} = \text{konst} < 0 \doteq -k^2$$

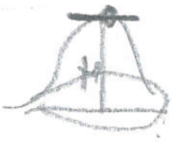
Tudjuk: $\boxed{f_2(\varphi) = A \cdot \sin(k(\varphi - \varphi_0))}$



Egész számú hullámhossz φ-ben: $\boxed{k = 2\pi n}$

$$\boxed{\frac{f_1''(r)}{f_1(r)} \cdot r^2 + \frac{f_1'(r)}{f_1(r)} r = -\frac{4\pi^2}{\lambda_0^2} + k^2 = -4\pi^2\left(\frac{1}{\lambda_0^2} - n^2\right)}$$

$f_1(0) = H$



$f_1'(0) = 0$

A megoldás nem elemi fgv (nem véges sin, cos, exp egyesítése?),

BESSEL FGV   Egzakt, Taylor sorokra

GOOGLE: vibrations of a circular membrane

FÜGGVÉNYEK GEOMETRIÁJA

$\Omega \subset \mathbb{R}^N$ korlátos, hozzá...

$\mathcal{F} := C(\bar{\Omega}) = \{\text{FOLYT } \bar{\Omega} \to \mathbb{R} \text{ fgv-ek}\}$

$$\langle f | g \rangle := \int_{x \in \Omega} fg\,dx = \int_{(x_1,..,x_N) \in \Omega} f(x_1,..,x_N)\, g(x_1,..,x_N)\,dx_1 \dots dx_N$$

SKALÁR-SZORZAT

Megjegyzés $\mathbb{R}^K$ HAGYOMÁNYOS $\langle | \rangle$-a:

$v = (v_1,..,v_K)$ FELFOGHATÓ EGY

$f : \{1,..,K\} \to \mathbb{R}$ $f(i) = v_i$ FGV-NEK

$$\langle f | g \rangle = \sum_{i=1}^{K} f(i) \cdot g(i)$$

$\sum$ HELYETT $\int$

$\mathcal{F}$ K-dim alterei a $d(f,g) = \sqrt{\langle f-g | f-g \rangle}$ távolsággal UAZ A GEOMETRIÁJA, MINT $\mathbb{R}^K$-NAK

Tétel A: $\Delta f = \mu f$, $f(\partial\Omega) = 0$, $f \in C^2(\Omega)$

SAJÁTFGV-EI A LAPLACE OP-NAK

ORTOGONÁLIS RSZ-T ALKOTNAK $\langle | \rangle$-RA

$\Delta f_1 = \mu_1 f_1$, $\Delta f_2 = \mu_2 f_2$ $\mu_1 \neq \mu_2 \Rightarrow \langle f_1 | f_2 \rangle = 0$

biz Emlékeztető: PARC $\int$ Ω fölött

1D: $\Omega = [a,b]$ $\quad \int_a^b f' g = fg\Big|_a^b - \int_a^b f g'$

Több D: $f' \rightsquigarrow \nabla f \quad g \rightsquigarrow [g_1, \dots, g_N] = G : \Omega \to \mathbb{R}^N$

Klasszikus: $\underbrace{\int_\Omega \langle \nabla f | G \rangle}_{\text{hagyományos } \mathbb{R}^N\text{-beli}} = \int_{\partial\Omega} \langle \underbrace{N_{\partial\Omega}}_{\text{normál vektor}} | f \cdot G \rangle - \int_\Omega f \cdot \underbrace{\mathrm{div}(G)}_{\sum_j \frac{\partial g_j}{\partial x_j}}$

DIFF FORMA: $\int_{\partial\Omega} \langle N_{\partial\Omega} | f \cdot G \rangle = \int_{\partial\Omega} f \cdot \sum_{j=1}^N g_j \, dx_{j+1} \wedge \dots \wedge dx_N \wedge dx_1 \wedge \dots \wedge dx_{j-1}$

MINT 3D-ben DIV, ROT-nál

$$\hat{G} := \sum_{j=1}^N g_j \, dx_{j+1} \wedge \dots \wedge dx_{j-1}$$

$$\int_\Omega \langle \nabla f | G \rangle = \int_\Omega \sum_{j=1}^N \frac{\partial f}{\partial x_j} \cdot g_j \, dx_1 \wedge \dots \wedge dx_N =$$

$$= \int_\Omega df \wedge \hat{G}$$

$$\int_\Omega f \cdot \mathrm{div}(G) = \int_\Omega f \, d\hat{G}$$

STOKES $\Rightarrow \int_{\partial\Omega} f \hat{G} = \int_\Omega d(f \cdot \hat{G}) =$

$$= \int_\Omega df \wedge \hat{G} + \int_\Omega f \, d\hat{G}$$

biz bef. Tf. $\Delta f = \mu f$, $\Delta g = \nu g$, $\mu \neq \nu$; $f, g : \partial\Omega \to 0$

$$\int_\Omega \langle \nabla f \mid \underbrace{\nabla g}_{G} \rangle = \int_{\partial\Omega} \langle N_{\partial\Omega} \mid \underset{\substack{\parallel \\ 0}}{f} \cdot \nabla g \rangle - \int_\Omega f \cdot \underbrace{\operatorname{div}(\nabla g)}_{\Delta g} =$$

$$= \int_\Omega f \cdot \nu g = \nu \int_\Omega f g = \nu \langle f \mid g \rangle$$

$f \leftrightarrow g$ csere $\Rightarrow \int_\Omega \langle \nabla f \mid \nabla g \rangle = \mu \langle f \mid g \rangle$

$\mu \neq \nu \Rightarrow \langle f \mid g \rangle = 0$ Q.e.d.

Megjegyzés $\int_{\partial\Omega} \langle N_{\partial\Omega} \mid f \cdot \nabla g \rangle = \int_{\partial\Omega} \langle N_{\partial\Omega} \mid g \nabla f \rangle = 0$

AKKOR IS, HA $\langle N_{\partial\Omega} \mid \nabla g \rangle = \langle N_{\partial\Omega} \mid \nabla f \rangle = 0$,

$\Rightarrow$ A SZABAD PEREMŰ REZGÉSEKHEZ TARTOZÓ ALAKFGV-EK IS ORTOGONÁLIS RSZT-ALKOTNAK