Fiz. elöch

1) Szuperpozíció: u_1, u_2 MEGOLD. $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$-NEK $\Rightarrow \alpha_1 u_1 + \alpha_2 u_2$ IS AZ

2) Kezdő beszolkozás helyett a mozgást meghatározzák a KEZDETI HELYEK ÉS SEBESSÉGEK

3) Általában 2)-höz adnak A KÜLSŐ ERŐK HATÁSAI

1) $\Rightarrow$ $\phi_1, \phi_2 : \mathbb{R} \to \mathbb{R}$ fgv, $\Rightarrow$ $\phi_1(x-ct) + \phi_2(x+ct)$ MEGOLD

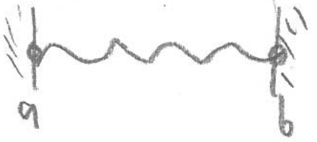
2)+1) HA $u(x,t)$ MEGOLD. és $u(x,0) = f(x)$, $\frac{\partial u}{\partial t}(x,0) = g(x)$,

AKKOR $u(x,t) = u_1(x,t) + u_2(x,t)$, AHOL

$u_1(x,0) = f(x)$, $\frac{\partial u_1}{\partial t}(x,0) = 0$;

$u_2(x,0) = 0$, $\frac{\partial u_2}{\partial t}(x,0) = g(x)$.

Fix végpontok esete



$0 = u(a,t) = u(b,t) = 0 \quad (t \in \mathbb{R})$

$u(x,t) = \phi(x-ct) + \Psi(x+ct)$

$u(a,t) = \phi(a - \underbrace{ct}_{x \text{ TETSZ}}) + \Psi(a+ct) = 0$

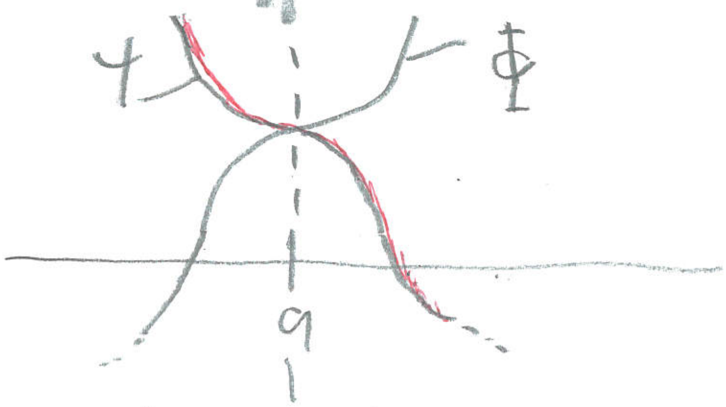
$\Psi(a+x) = -\phi(a-x) \quad (x \in \mathbb{R})$!!! HIÁBA VÉGES A HÚR

$\phi, \Psi : \mathbb{R} \to \mathbb{R}$

$x \rightsquigarrow x-a$

$$\boxed{\Psi(x) = \phi(a-(x-a)) = \phi(2a-x)}$$

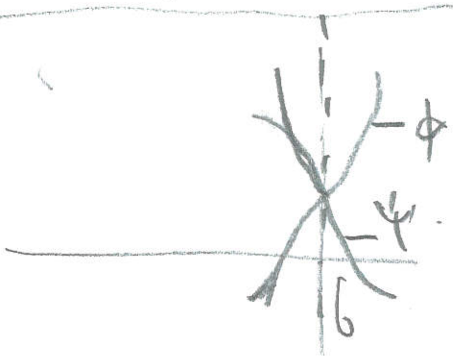
Geometriailag



$$\text{grafikon}(\Psi) = \underbrace{= \text{TÜKR}_a}_{T_a}\ \text{grafikon}(\phi)$$

HASONLÓAN $u(b,t) \equiv 0 \implies$

$$\boxed{\Psi(x) = \phi(2b-x)}$$



$$\text{grafikon}(\Psi) = \underbrace{= \text{TÜKR}_b}_{T_b}\ \text{grafikon}(\phi)$$

$$\Psi = T_a(\phi) = T_b(\phi) \quad / T_a \qquad T_a^2 = Id$$

$$\phi = T_a T_b(\phi)$$

$$\phi(x) = T_a[\phi(2b-x)] =$$
$$= \phi(2a-(2b-x)) =$$
$$= \phi(2(a-b)+x)$$

$T_a T_b$: eltolás $2(a-b)$-vel

ϕ, Ψ periodikus $\underbrace{2(a-b)}_{=L}$-re

Kell-e további feltétel ?

Tf. $\phi : \mathbb{R} \to \mathbb{R}$ per $2L$-re $\quad \phi(x + n \cdot 2L) = \phi(x)$

$n = 0, \pm 1, \pm 2, \dots$

$\Psi(x) = \phi(2a - x) \qquad L = b - a$

$$\phi(a-ct) - \Psi(a+ct) = \phi(a-ct) - \phi(2a-(a+ct)) =$$
$$= \phi(a-ct) - \phi(a-ct) \equiv 0$$

$$\phi(b-ct) - \Psi(b+ct) = \phi(b-ct) - \phi(2a-(b+ct)) =$$
$$\underset{\uparrow \phi \text{ 2L-PER}}{=} \phi(b - ct \underset{2b-2a}{-2L}) - \phi(2a - b - ct) =$$
$$= \phi(2a-b-ct) - \phi(2a-b-ct) \equiv 0$$

<u>Tétel</u> $u(x,t) = \phi(x-ct) - \Psi(x+ct)$ egy a,b-ben rögz. húr mozgása, ha

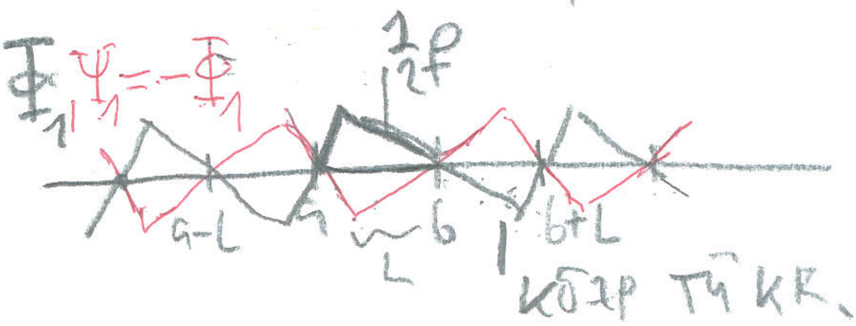
$\phi : \mathbb{R} \to \mathbb{R}$ $2(b-a)$-periodikus és

$\Psi(x) \equiv \phi(2a - x)$ [$\equiv \phi(2b-x)$ is igaz].

<u>Gitár</u> $u_1(x,0) = f(x) \quad (a \le x \le b) \quad f(a) = f(b) = 0$

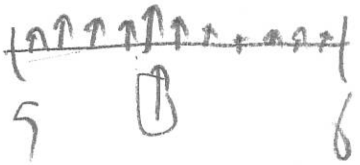
$\frac{\partial u_1}{\partial t}(x,0) \equiv 0 \qquad$ [0 sebességgel rendítés]

$u_1(x,t) = \phi_1(x-ct) - \Psi_1(x+ct)$:



Zongora $\quad u_2(x,0)=0 \quad \frac{\partial u_2}{\partial t}(x,0)=g(x) \quad a\le x\le b$

$g(a)=g(b)=0$ [alaphelyzetű húrra ütünk a kalapáccsal, x-nél g(x) kezdősebességet okozva]



$$u_2(x,t)=\phi_2(x-ct)-\Psi_2(x+ct)$$

$$0=u_2(x,0)=\phi_2(x)-\Psi_2(x) \Rightarrow \phi_2=\Psi_2 \quad \text{végtelen nagyságú}$$

$$g(x)=\frac{\partial u_2}{\partial t}(x,0)=\frac{\partial}{\partial t}\Big|_{t=0}\left\{\phi_2(x-ct)-\phi_2(x+ct)\right\}\Big|_{[a,b]\text{-n}}$$

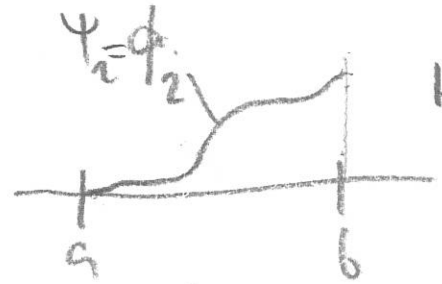
$$=\phi_2'(x)\cdot(-c)-\phi_2'(x)\cdot c=$$

$$=-2c\cdot\phi'(x)$$

$$\phi_2'(x)=-\frac{1}{2c}g(x) \qquad a\le x\le b$$

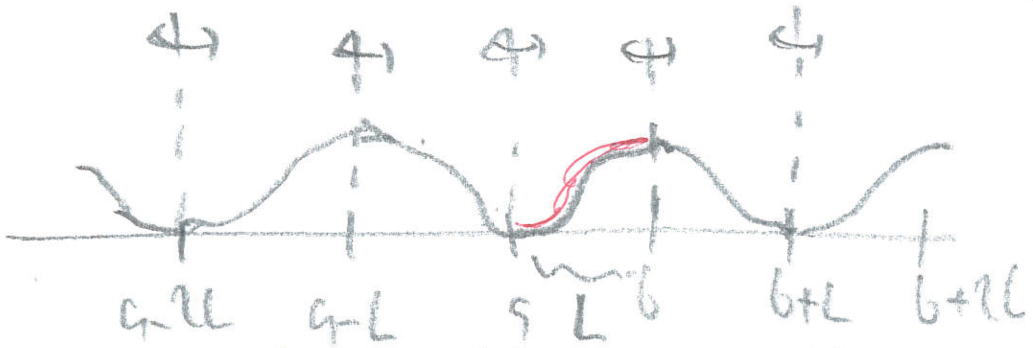
$$\phi_2(x)=\int_a^x \phi_2'(\xi)\,d\xi=-\int_a^x\frac{1}{2c}g(\xi)\,d\xi \qquad a\le x\le b$$

$$\underset{\Psi_2(x)}{\parallel} \qquad \underset{\phi_2(a)=0}{\uparrow}$$



HOGYAN TOVÁBB ???

TÜKRÖZÉSEK

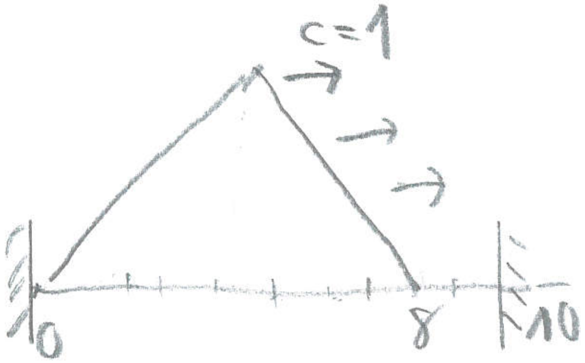


$$u_2(x,t) = \phi_2(x-ct) - \phi_2(x+ct)$$

PERIODICITÁS $2L/c = 2(b-a)/c$ IDŐNKÉNT

<u>Példa</u> Visszaverődés

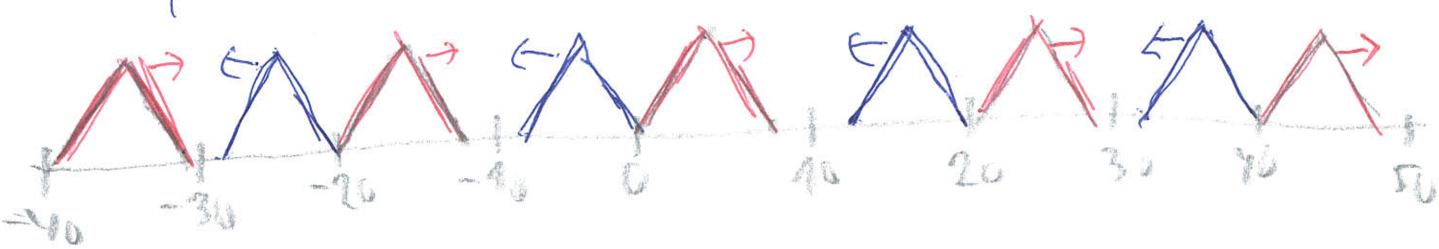
Kezdőállapot $t=0$

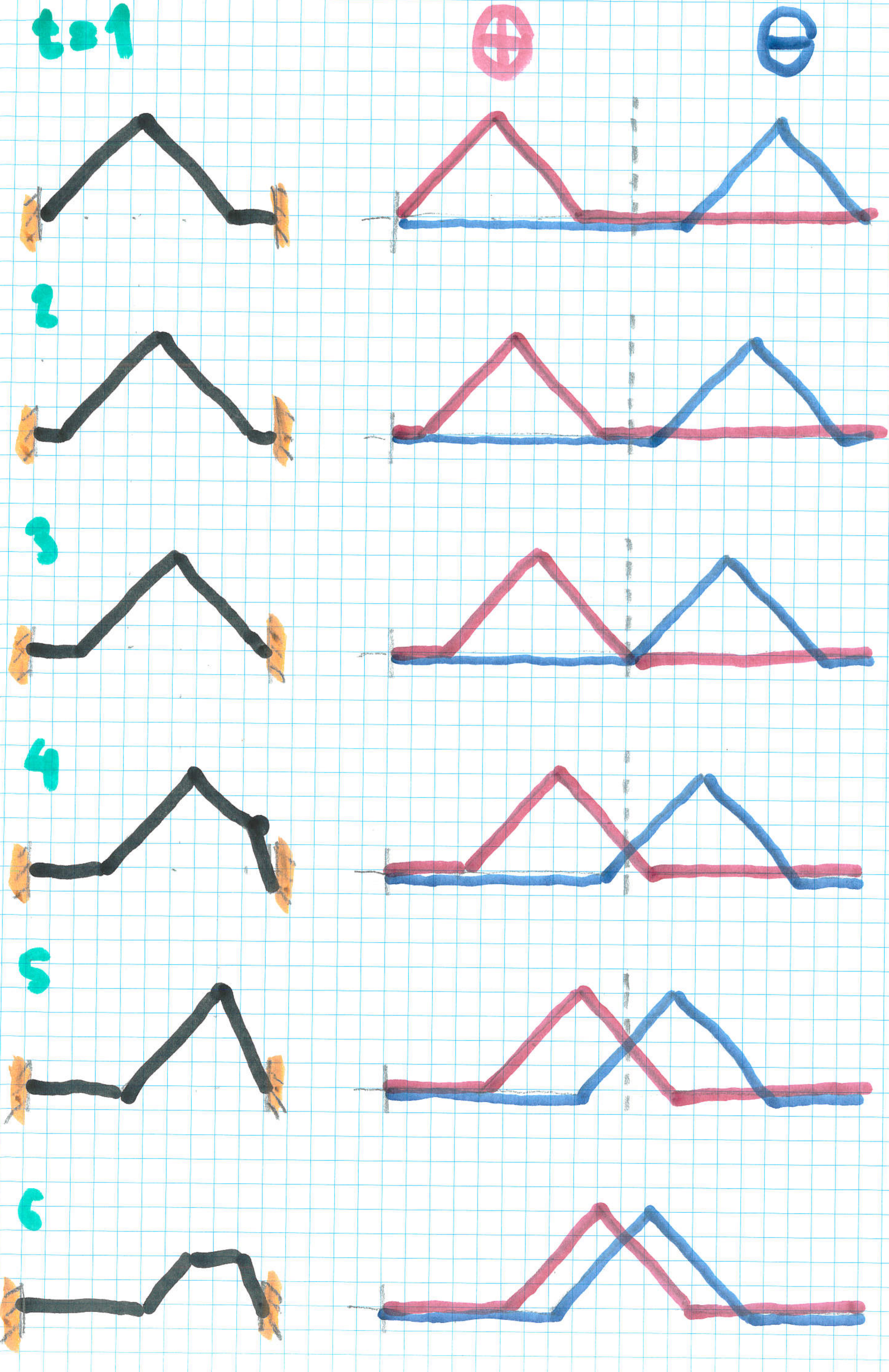


HF) $u(x,t) = \underbrace{u_1(x,t)}_{\text{GITÁR}} + \underbrace{u_2(x,t)}_{\text{ZONGORA}}$

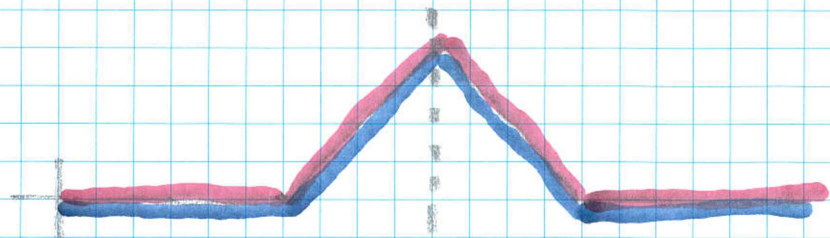
$$u(x,t) = \phi(x,t) - \Psi(x,t)$$

$\phi(x)$, $\Psi(x)$




t=1
2
3
4
5
6
⊕
⊖

7



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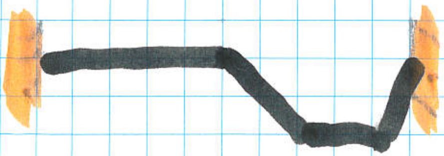

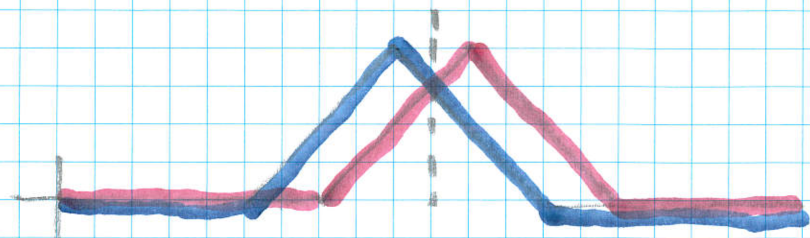

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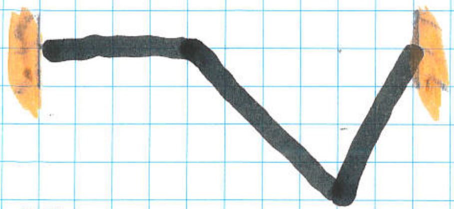

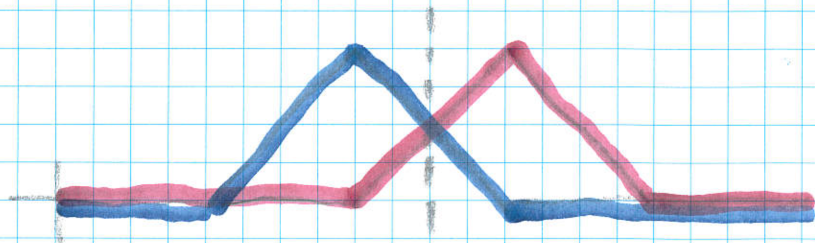

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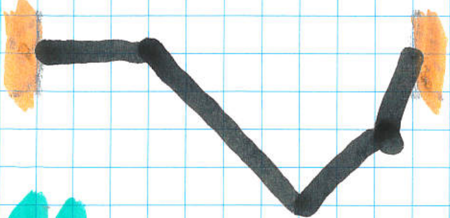

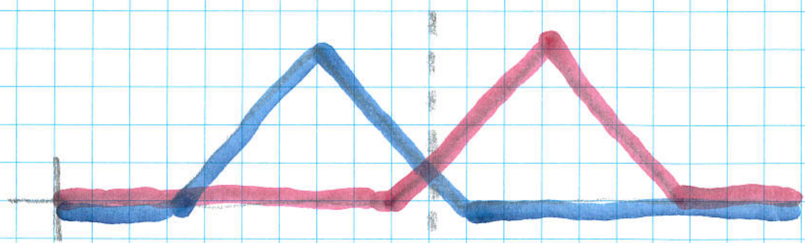

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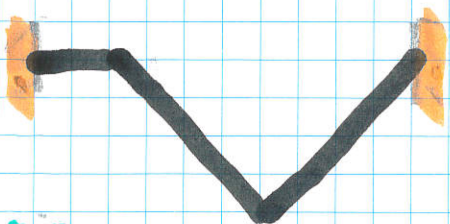

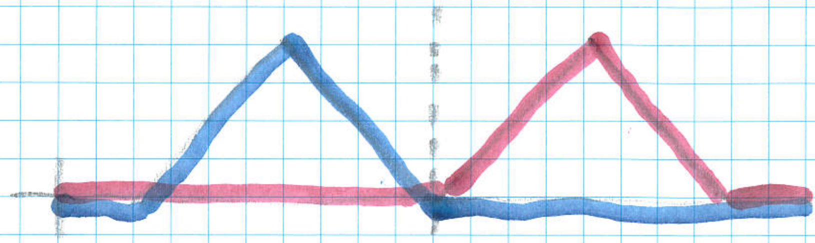

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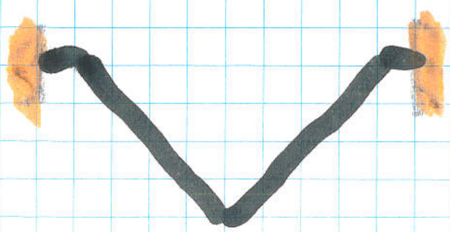
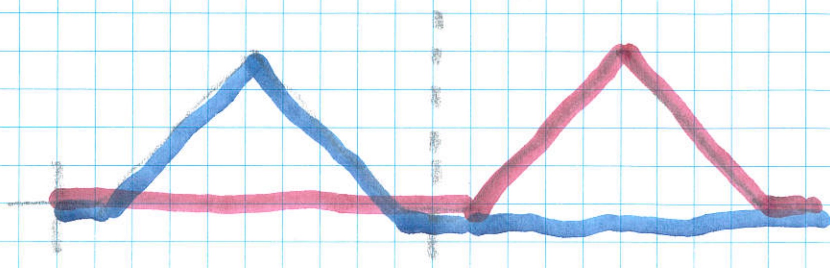




Tétel Tekintsük $u(a,t)=p(t)$, $u(b,t)=q(t)$ $(t\in\mathbb{R})$

$$u(x,0)=f(x),\ \frac{\partial u}{\partial t}(0,x)=g(x) \qquad (x\in[a,b])$$

ELLENTMONDÁS-MENTES kezdeti feltétel mellett $\exists!$ $u(x,t)$

$$u:[a,b]\times\mathbb{R}\to\mathbb{R} \quad \text{MEGOLDÁS}$$

Biz Szuperpozíció $\Longrightarrow$ $u=u_1+u_2+u_3+u_4$

ahol u_1 GITÁR, u_2 ZONGORA $\equiv 0$

$$\left.\begin{array}{l} u_3(a,t)=0,\ u_3(b,t)=q(t) \\ u_4(a,t)=p(t),\ u_4(b,t)=0 \end{array}\right\} t\in\mathbb{R}$$

1) u_3 KONSTR.

$$u_3(a,t)=\phi_3(x-ct)-\psi_3(x+ct)$$

$$0=u_3(a,t)=\phi_3(\underbrace{a-ct}_{2a-x})-\psi_3(\underbrace{a+ct}_{x})$$

$$\psi_3(x)=\phi_3(2a-x)$$

$$q(t)=u_3(b,t)=\phi_3(b-ct)-\psi_3(b+ct)=$$

$$=\phi_3(b-ct)-\phi_3(2a-(b+ct))=$$

$$=\phi_3(\underbrace{b-ct}_{x})-\phi_3(\underbrace{b-2L-ct}_{x}) \quad \text{AHOL } L=b-a$$

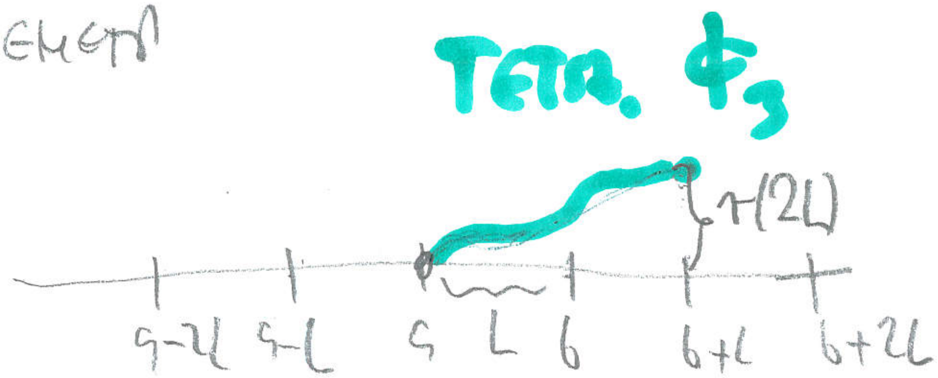
$[a,b]$ HOSSZA

$$x = b - ct \Rightarrow t = \frac{b-x}{c}$$

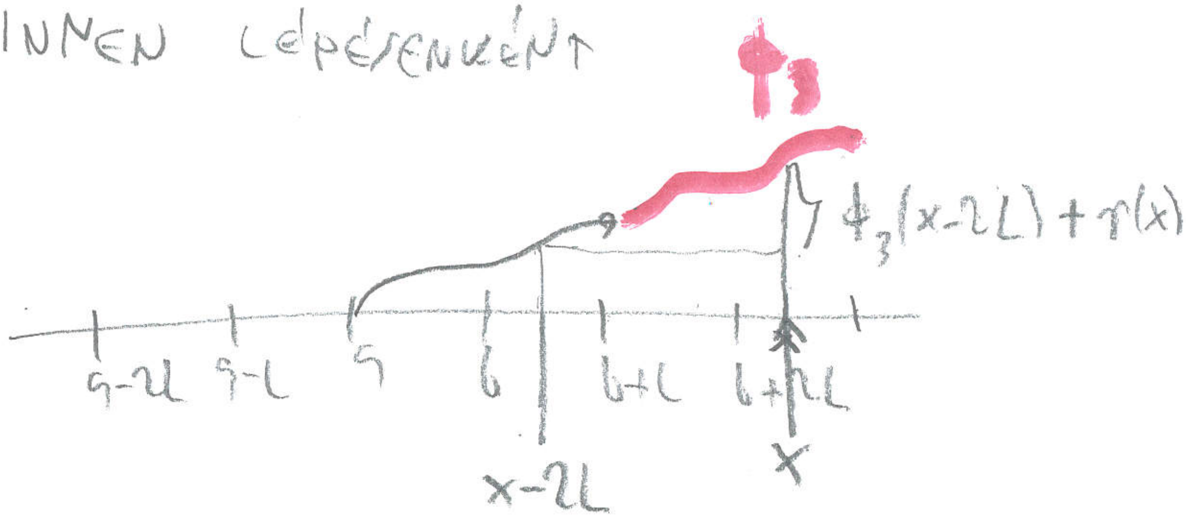
$$\phi_3(x) - \phi_3(x-2L) = g\left(\frac{b-x}{c}\right)$$

$$\boxed{\phi_3(x-2L) = \phi_3(x) - r(x)}$$ AHOL $r(x) := g\left(\frac{b-x}{c}\right)$

VEHETŐ



INNEN LÉPÉSENKÉNT



BALRA IS HASONLÓAN

2) UGYANÍGY $(a \Leftrightarrow b)$ u_4 KONSTRUKCIÓJA

3) u_3, u_4 ALAPJÁN u_1 AZ

$$u_1(x,0) = f(x) - u_3(x,0) - u_4(x,0) \quad (x \in [a,b])$$

KEZDŐFELTÉTELLEL