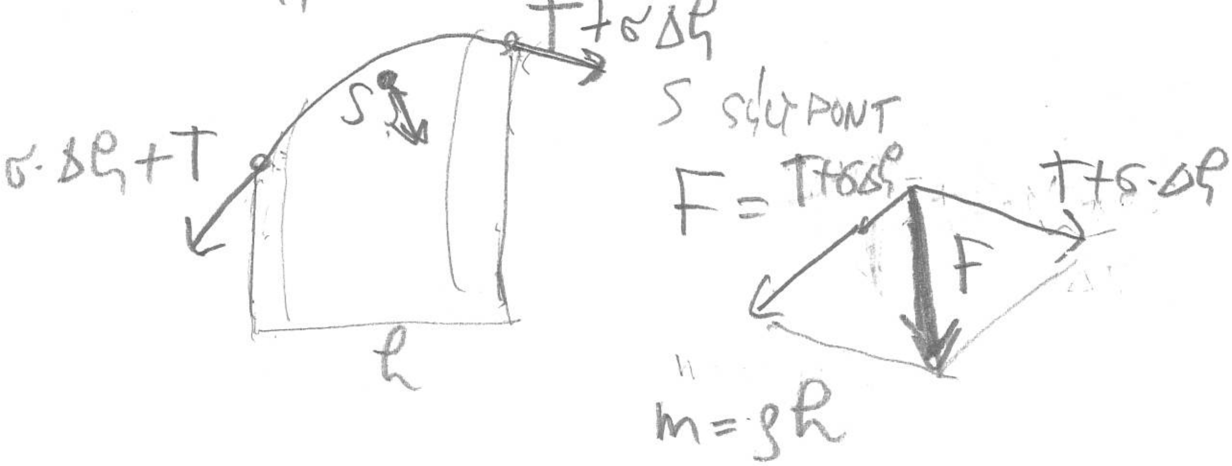
HULLÁMEGYENLET

1D) Homogén rezgő húr

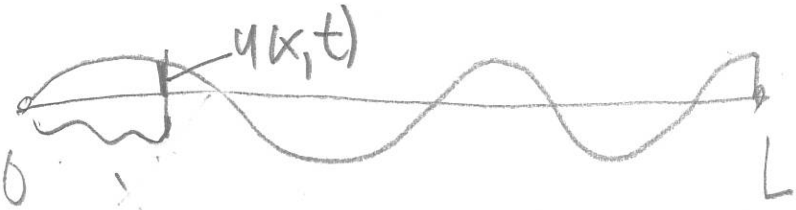
T feszítőerő (mindenütt ua)

tömeg = hossz · ρ – nyugalomnál

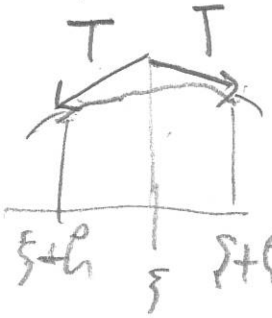
$F = ma$



LINEÁRIS KÖZELÍTÉS KIS KITÉRÉSNÉL



t időpillanatban



FERDESÉG ELHANYAGOLVA

Δh ELHANYAGOLVA

$\approx F \perp$ alap

$$\left.\frac{\partial u}{\partial x}\right|_{x=\xi+\rho} - \left.\frac{\partial u}{\partial x}\right|_{x=\xi-\rho}$$

MEREDEKSÉG

$\operatorname{tg}\alpha = \frac{1}{2}(\ )$

$$F \approx 2T\sin\alpha \underset{\substack{\uparrow \\ \text{KIS SZÖGEKRE}}}{\approx} 2T\,\mathrm{tg}\,\alpha = 2T\left(\frac{\partial y}{\partial x}\Big|_{x=\xi+h} - \frac{\partial y}{\partial x}\Big|_{x=\xi-h}\right)/2$$

$$m = \rho \cdot 2h$$

$\updownarrow$s GYORSULÁSA $= \dfrac{\partial^2 y}{\partial t^2}\Big|_{x=\xi}$

$$ma \approx F$$

$$2\rho h \cdot \frac{\partial^2 y}{\partial t^2} \approx T \cdot \left(\frac{\partial y}{\partial x}\Big|_{x=\xi+h} - \frac{\partial y}{\partial x}\Big|_{x=\xi-h}\right) \quad /:(2h)$$

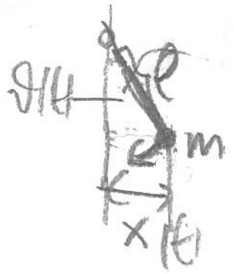
$h \to 0$

$$\boxed{\rho \frac{\partial^2 y}{\partial t^2} = T \cdot \frac{\partial^2 y}{\partial x^2}}$$

HULLÁMEGYENLET
IDEALIZÁLT

HF: 1) HA $\rho = \rho(x)$, AKKOR IS $\rho \dfrac{\partial^2 y}{\partial t^2} = T \cdot \dfrac{\partial^2 y}{\partial x^2}$

2) INGA



$$m\frac{\partial^2 x}{\partial t^2} = -mg\sin\vartheta(t)\cos\vartheta(t)$$

$$= -mg\frac{x(t)}{\ell}\sqrt{1-\left(\frac{x(t)}{\ell}\right)^2} \approx$$

$$\approx -mg\frac{x(t)}{\ell}\left[1-\frac{1}{2}\left(\frac{x(t)}{\ell}\right)^2\right] \underset{\substack{\uparrow \\ \text{1 REND}}}{\approx}$$

$$\approx -mg\frac{x(t)}{\ell}$$

D' Alembert megoldás

$$\rho \frac{\partial^2 u}{\partial t^2} = T \frac{\partial^2 u}{\partial x^2}$$

$$\frac{T}{\rho} = \frac{\text{erő}}{\text{tömeg}/\text{hossz}} = \frac{\text{tömeg}\cdot\text{hossz}/\text{idő}^2}{\text{tömeg}/\text{hossz}} = \frac{\text{hossz}^2}{\text{idő}^2} =$$

$$= \text{sebesség}^2 = c^2 \qquad \text{terjedési sebesség}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(*) $\left.\begin{matrix} y = x + ct \\ z = x - ct \end{matrix}\right\}$ $X = \begin{bmatrix} x \\ t \end{bmatrix}$ $Y = \begin{bmatrix} y \\ z \end{bmatrix}$ $\quad \frac{\partial u}{\partial X} = \frac{\partial u}{\partial Y}\frac{\partial Y}{\partial X}$

$$\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix} \begin{bmatrix} \partial y/\partial x & \partial y/\partial t \\ \partial z/\partial x & \partial z/\partial t \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix} \begin{bmatrix} 1 & c \\ 1 & -c \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \qquad \frac{\partial f}{\partial t} = c\frac{\partial f}{\partial y} - c\frac{\partial f}{\partial t}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial(\partial u/\partial x)}{\partial x} = \frac{\partial(\partial u/\partial x)}{\partial y} + c\frac{\partial(\partial u/\partial x)}{\partial z} =$$

$$= \frac{\partial}{\partial y}\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right] = \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial^2 u}{\partial t^2} = HF = c^2\left[\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 u}{\partial z^2}\right]$$

$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ az $Y = \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} x+ct \\ x-ct \end{bmatrix}$ KOORD RSZ-BEN

$$c^2\left[\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial^2 u}{\partial y\,\partial z} + \frac{\partial^2 u}{\partial z^2}\right] = c^2\cdot\left[\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial y\,\partial z} + \frac{\partial^2 u}{\partial z^2}\right]$$

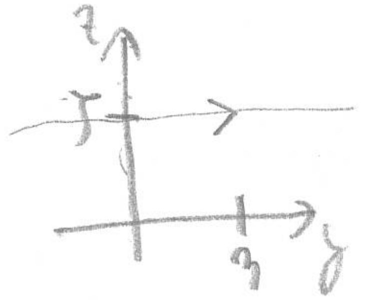
$$\boxed{0 = 4\cancel{c^2}\,\frac{\partial^2 u}{\partial y\,\partial z}}$$

__Tétel__ $u:\mathbb{R}^2\to\mathbb{R}$ 2x folyt. diff, $\begin{bmatrix} y\\ z\end{bmatrix}: \mathbb{R}^2\leftrightarrow\mathbb{R}^2$ KOORD (LIN)

$$\frac{\partial^2 u}{\partial y\,\partial z}=0 \Rightarrow \boxed{u(y,z)\equiv \Phi(y)+\Psi(z)\cancel{-u(0,0)}}$$

$\Phi(\eta)=u(\eta,0)$, $\Psi(\zeta)=u(0,\zeta)$

Biz $\frac{\partial}{\partial y}\frac{\partial u}{\partial z}=0$



FIX ζ

$\frac{d}{d\tau}\frac{\partial u}{\partial z}(\tau,\zeta)=0$

$\tau\mapsto \frac{\partial u}{\partial z}(\tau,\zeta)\equiv \text{const}(\zeta) =$

$= \frac{\partial u}{\partial z}(0,\zeta) = \frac{d}{d\zeta}u(0,\zeta)$

$\frac{\partial u}{\partial z}(\tau,\zeta) = \Psi'(\zeta)$ ahol $\underline{\Psi(\zeta) := u(0,\zeta)}$

$\frac{\partial u}{\partial y}(\eta,\tau)=\Phi'(\eta)$, $\underline{\Phi(\eta):=u(\eta,0)}$

$u(\eta,\zeta)=u(\eta,0)+[u(\eta,\zeta)-u(\eta,0)] =$

$=\Phi(\eta)+\int_{\tau=0}^{\zeta}\frac{\partial u}{\partial z}(\eta,\tau)\,d\tau$

$$u(\eta,\xi) = \phi(\eta) + \int_{\tau=0}^{\eta} \psi'(\tau)\, d\tau =$$

$$= \phi(\eta) + [\Psi(\eta) - \underbrace{\Psi(0)}_{u(0,0)}]$$

<u>D'Alembert tétel</u> $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ MEGOLDÁSAI

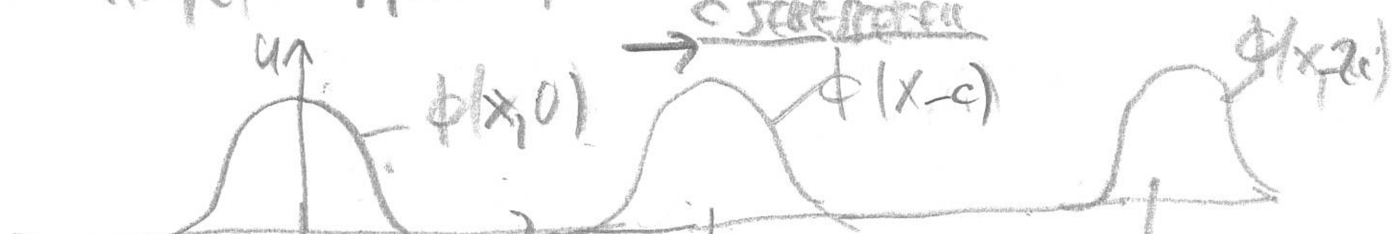
$$\boxed{u(x,t) = \Phi(x+ct) + \Psi(x-ct)}$$

<u>Megjegyzés</u> ÁLT. ÉRT. DERIVÁLTAKKAL ($\rightarrow$ DISZTRIBÚCIÓK)

ϕ, Ψ TETSZ. FOLYT. FGV IS LEHET

<u>Haladó hullámok</u>

$$u(x,t) = \phi(x-ct)$$



HÁTRA HALADÓ $u(x,t) = \Psi(x+ct)$

„Filmkockák"