DIFF-FORMA PRIMITÍVEI

Megj $f \in C(\mathbb{R})$ esetén $F \in C^1(\mathbb{R})$ f primitív függv., ha $F' = f$.

Diff formákkal: F (mint 0-forma) primitívje $f\,dx$-nek, ha $dF = f\,dx$

$F_0 := \left[\xi \mapsto \int_{t=0}^{\xi} f(t)\,dt \right]$ mellett $\{f$ primit. fgv-ei$\} = \{F_0 + c : c \in \mathbb{R}\}$

Def $\omega : \mathbb{R}^N \to \{$alternáló K-formák$\}$ primitívje $\Omega : \mathbb{R}^N \to \{$alt. $(K-1)$-formák$\}$ ha $d\Omega = \omega$. $\underbrace{\Omega_0(x)}_{[v_1, \dots, v_{k-1}]} := \int_{t=0}^{1} t^{k-1} \omega(tx)[x, v_1, \dots, v_{k-1}]\,dt$

Tétel $P_\omega = \{\omega$ primitívjei$\} \neq \emptyset \Leftrightarrow d\omega = 0$, ilyenkor $P_\omega = \{\Omega_0 + dC : C$ $(k-2)$-forma$\}$

Biz 1) $\Omega \in P_\omega \Rightarrow d\Omega = \omega \Rightarrow 0 = d^2\Omega = d\omega$

2) $\Leftarrow$ stratégia: ω-t meghatározza $Q \mapsto \int_Q \omega$ (Q k dim C^1 kocka)

3) Q-hoz $\tilde{Q} : [0,1]^{k+1} \ni (t, t_1, \dots, t_k) \mapsto t\,Q(t_1, \dots, t_k)$

$\underline{\text{Áll}}$ Ha $\partial Q = \sum_{m=1}^{2k} R_m$ ahol $R_1, \dots, R_n$ $(k-1)$-formák,

akkor $\partial \tilde{Q} = Q + \sum_{m=1}^{2k} \tilde{R}_m$

4) Feltevés $d\omega = 0$ feltevés

Stokes $\Rightarrow$ $0 = \int_{\tilde{Q}} d\omega = \int_{\partial \tilde{Q}} \omega = \int_Q \omega + \sum_{m=1}^{2k} \int_{\tilde{R}_m} \omega$

$$\int_Q \omega = -\sum_{m=1}^{2k} \int_{\tilde{R}_m} \omega$$

5) $R = (s_1,..,s_{k-1}) \mapsto R(s_1,..,s_{k-1})$ (k-1)-di Ck-hoch

$\tilde{R} = (t, s_1,..,s_{k-1}) \mapsto t R(s_1,..,s_{k-1})$ k-di Ck-hoch

Tvdjik: $\int_{\tilde{R}} \omega = \int_{(t,s_1,..,s_{k-1}) \in [0,1]^k} \omega(\tilde{R}(t,s_1,..,s_{k-1})) \left[\frac{\partial \tilde{R}}{\partial t}, \frac{\partial \tilde{R}}{\partial s_1},.., \frac{\partial \tilde{R}}{\partial s_{k-1}}\right] dt\, ds_1 .. ds_{k-1}$

$$\int_{\tilde{R}} \omega = \int_{t=0}^{1} \int_{s \in [0,1]^{k-1}} \omega(t R(s)) \left[R, t\frac{\partial R}{\partial s_1},\dots, t\frac{\partial R}{\partial s_{k-1}}\right] ds_1 .. ds_{k-1}\, dt =$$

$$= \int_{t=0}^{1} t^{k-1} \omega(tx) \left[x, \frac{\partial R}{\partial s_1},.., \frac{\partial R}{\partial s_{k-1}}\right] ds_1 .. ds_{k-1}\, dt =$$

$$= \int_R \Omega_0$$

6) $k \sigma u$: $\int_Q \omega = \sum_{n=1}^{2k} \int_{R_{k,n}} \Omega_0 = \int_{\partial Q} \Omega_0 = \int_Q d\Omega_0$ Q.e.d.

Stokes

$\{0$ prinlg...$\} = \{dC = C(k-1)\dots\}$

<u>Koordinaten</u>

$$\omega = \sum_{1 \le i_1 < \dots < i_k \le N} f_{i_1,..,i_k} \underbrace{dx_{i_1} \wedge \dots \wedge dx_{i_k}}_{\omega_I} = \sum_I f_I\, dx_I, \quad I = (i_1,..,i_k)$$

$$\Omega_I = \int_{t=0}^{1} t^{k-1} \omega_I(tx) [x, v_1,.., v_{k-1}]\, dt$$

$$\int_R \Omega_I = \int_{t=0}^{1} t^{k-1} \int_{s \in [0,1]^{k-1}} f_I(t R(s))\, \det \begin{bmatrix} R_{i_1}(s) & \frac{\partial R_{i_1}}{\partial s_1} & \dots & \frac{\partial R_{i_1}}{\partial s_{k-1}} \\ \vdots & \vdots & & \vdots \\ R_{i_k}(s) & \frac{\partial R_{i_k}}{\partial s_1} & \dots & \frac{\partial R_{i_k}}{\partial s_{k-1}} \end{bmatrix} ds\, dt$$

$$\Omega_I = \int_{t=0}^{1} t^{k-1} f_I(tx) \sum_{j=1}^{k} (-1)^{j-1} x_{i_j}\, dx_{i_1} \wedge \dots \wedge \widehat{dx_{i_j}} \wedge \dots \wedge dx_{i_k}$$

Példa: $w = \frac{1}{1+x^2+y^2+4z^2}\left(y\,dy\wedge dz + x\,dx\wedge dz\right)$ PRIMITÍVJE

$$dw = \frac{\partial[y/(1+x^2+y^2+4z^2)]}{\partial x}\,dx\wedge dy\wedge dz +$$

$$+\frac{\partial[x/(1+x^2+y^2+4z^2)]}{\partial y}\,\underbrace{dy\wedge dx\wedge dz}_{-dx\wedge dy\wedge dz} =$$

$$= -2\frac{xy}{(1+x^2+y^2+4z^2)^2}\,dx\wedge dy\wedge dz + (-2)\frac{xy}{(1+x^2+y^2+4z^2)^2}\,\underbrace{dy\wedge dx\wedge dz}_{-dx\wedge dy\wedge dz} = \underline{\underline{0}}$$

$$\Omega = \int_{t=0}^{1} t\,\frac{ty}{1+t^2x^2+t^2y^2+4t^2z^2}\,[\underbrace{y\,dz - z\,dy}_{dy\wedge dz}]\,dt +$$

$$+\int_{t=0}^{1} t\,\frac{tx}{1+t^2x^2+t^2y^2+4t^2z^2}\,[x\,dz - z\,dx]\,dt =$$

$$= \int_{t=0}^{1}\frac{t^2\,dt}{1+t^2x^2+t^2y^2+4t^2z^2}\left[(y^2+x^2)\,dz - yz\,dy - xz\,dx\right]$$

$\frac{1}{1+x^2+y^2+4z^2}$ HELYETT CSAK $(1+x^2+y^2+4z^2)$