$$\frac{\partial f}{\partial p} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial p} \qquad \begin{bmatrix}\frac{\partial f}{\partial r} & \frac{\partial f}{\partial \varphi}\end{bmatrix} = \begin{bmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{bmatrix}\begin{bmatrix}\cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi\end{bmatrix}$$

KELL: $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ KIFEJEZÉSE r, φ-VEL

$$\begin{bmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{bmatrix} = \begin{bmatrix}\frac{\partial f}{\partial r} & \frac{\partial f}{\partial \varphi}\end{bmatrix}\begin{bmatrix}\cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi\end{bmatrix}^{-1} =$$

$$= \begin{bmatrix}\frac{\partial f}{\partial r} & \frac{\partial f}{\partial \varphi}\end{bmatrix}\begin{bmatrix}\cos\varphi & \sin\varphi \\ -\frac{1}{r}\sin\varphi & \frac{1}{r}\cos\varphi\end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\cdot\cos\varphi - \frac{\partial f}{\partial \varphi}\cdot\frac{1}{r}\sin\varphi \qquad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial r}\sin\varphi + \frac{\partial f}{\partial \varphi}\cdot\frac{1}{r}\cos\varphi$$

$\frac{\partial^2}{\partial x^2}$, $\frac{\partial^2}{\partial y^2}$ KIF r, φ-vel

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial g}{\partial x} \quad \text{AHOL} \quad g = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\cos\varphi - \frac{\partial f}{\partial \varphi}\cdot\frac{1}{r}\sin\varphi$$

$$\|$$

$$\frac{\partial g}{\partial r}\cos\varphi - \frac{\partial g}{\partial \varphi}\cdot\frac{1}{r}\sin\varphi =$$

$$= \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\cos\varphi - \frac{\partial f}{\partial \varphi}\cdot\frac{1}{r}\sin\varphi\right)\right]\cos\varphi - \left[\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\cos\varphi - \frac{\partial f}{\partial \varphi}\cdot\frac{1}{r}\sin\varphi\right)\right]\frac{1}{r}\sin\varphi$$

$$= \left[\frac{\partial^2 f}{\partial r^2}\cos\varphi - \frac{\partial^2 f}{\partial r\partial\varphi}\frac{1}{r}\sin\varphi + \frac{\partial f}{\partial \varphi}\frac{1}{r^2}\sin\varphi\right]\cos\varphi -$$

$$- \left[\frac{\partial^2 f}{\partial\varphi\partial r}\cos\varphi - \frac{\partial f}{\partial r}\sin\varphi - \frac{\partial^2 f}{\partial\varphi^2}\cdot\frac{1}{r}\cos\varphi - \frac{\partial f}{\partial \varphi}\cdot\frac{1}{r}\cos\varphi\right]\frac{1}{r}\sin\varphi =$$

$$= \frac{\partial^2 f}{\partial r^2}\cos^2\varphi + \underset{\text{15}}{\frac{\partial^2 f}{\partial r\partial\varphi}}\left(-2\frac{1}{r}\cos\varphi\sin\varphi\right) + \frac{\partial^2 f}{\partial\varphi^2}\cdot\frac{1}{r}\cos\varphi + \frac{\partial f}{\partial r}\cdot\frac{1}{r}\sin^2\varphi + \frac{\partial f}{\partial \varphi}\frac{1}{r}\sin\varphi\cos\varphi$$

HF: $\frac{\partial^2 f}{\partial y^2}$ HASONLÓAN

Eredmény: $\boxed{\Delta f = \frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial r^2}+\frac{1}{r}\frac{\partial f}{\partial r}+\frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2}}$

DIFF FORMA GEOMETRIAILAG

<u>Def</u> S absztrakt halmaz (felület)

$f, u_1, \dots, u_k : S \to \mathbb{R}$

$Q : [a_1, b_1] \times \dots \times [a_k, b_k] \to S$ ($\sim$ k-dim felület S-ben)

$$\int_Q f\, du_1 \wedge \dots \wedge du_k := \int_{t_1=a_1}^{b_1} \dots \int_{t_k=a_k}^{b_k} f(Q(t_1, \dots, t_k)) \cdot$$

$$\cdot \det\left[\frac{\partial u_i(Q(t_1, \dots, t_k))}{\partial t_j}\right] dt_k \dots dt_1$$

<u>T</u> Ha $X : S \to \mathbb{R}^N$ olyan koord, hogy ${}^X f, {}^X u_i, {}^X x_j(Q)$ f. diff

akkor
$$\int_Q f\, du_1 \wedge \dots \wedge du_k = \int_{{}^X Q} {}^X f({}^X Q(t_1, \dots, t_k))\, d\,{}^X u_1 \wedge \dots \wedge d\,{}^X u_k$$

ahol ${}^X Q : (t_1, \dots, t_k) \mapsto \begin{bmatrix} x_1(Q(t_1, \dots, t_k)) \\ \vdots \\ x_N(Q(t_1, \dots, t_k)) \end{bmatrix}$

$$d\,{}^X u_i = \frac{\partial\, {}^X u_i}{\partial x_1} dx_1 \wedge \dots \wedge \frac{\partial\, {}^X u_i}{\partial x_N} dx_N$$

Példa $\quad S = Pol(\mathbb{R}) = \{ \mathbb{R} \to \mathbb{R} \text{ polinomok} \}$

$$u_1(p) := \int_{z=0}^{2} p(z)\,dz \qquad u_2(p) := \int_{z=1}^{3} p(z)\,dz \qquad f(p) := \int_{z=2}^{4} p(z)\,dz$$

$$Q : [0,1]^2 \to S \qquad Q(t_1,t_2) = (t_1 \cdot z)^3 + 3(t_2 z)^2$$

$$\int_Q f\, du_1 \wedge du_2 = ?$$

$$\int_Q f\, du_1 \wedge du_2 = \int_{t_1=0}^{1} \int_{t_2=0}^{1} f(p) \det \begin{bmatrix} \partial u_1/\partial t_1 & \partial u_1/\partial t_2 \\ \partial u_2/\partial t_1 & \partial u_2/\partial t_2 \end{bmatrix} \Bigg|_{p=Q(t_1,t_2)} dt_2\, dt_1 =$$

$$f(p)\Big|_{p=Q(t_1,t_2)} = \int_{z=2}^{4} Q(t_1,t_2)\,dz = \int_{z=2}^{4} \left(t_1^3 z^3 + 3t_1^2 z^2\right) dz =$$

$$= \left(\int_{z=2}^{4} z^3\,dz\right) t_1^3 + \left(3\int_{z=2}^{4} z^2\,dz\right) t_2^2 = 60\, t_1^3 + 56\, t_1^2$$

$$u_1(p)\Big|_{p=Q(t_1,t_2)} = \left[\text{HASONLÓ } \int_{z=0}^{2}\right] =$$

$$= \left(\int_{z=0}^{2} z^3\,dz\right) t_1^3 + \left(3\int_{z=0}^{2} z^2\,dz\right) t_2^2 = 4\,t_1^3 + 8\,t_2^2$$

$$u_2(p)\Big|_{p=Q(t_1,t_2)} = \left[\text{HASONLÓ } \int_{z=1}^{3}\right] = 20\, t_1^3 + 56\, t_2^2$$

$$\det\left[\frac{\partial u_i}{\partial t_j}(p)\right]\Bigg|_{p=Q(t_1,t_2)} = \det\begin{bmatrix} \frac{\partial}{\partial t_1}(4t_1^3+8t_2^2) & \frac{\partial}{\partial t_2}(4t_1^3+8t_2^2) \\ \frac{\partial}{\partial t_1}(20t_1^3+26t_2^2) & \frac{\partial}{\partial t_2}(20t_1^3+26t_2^2) \end{bmatrix} =$$

$$= \det\begin{bmatrix} 12\,t_1^2 & 16\,t_2 \\ 60\,t_1^2 & 52\,t_1 \end{bmatrix} = \det\begin{bmatrix} 12 & 16 \\ 60 & 52 \end{bmatrix} t_1^2 t_2 = -336\, t_1^2 t_2$$

$$\int_Q f\, du_1 \wedge du_2 = \int_{t_1=0}^{1}\int_{t_2=0}^{1} (60t_1^3+56t_2^2)\cdot(-336\, t_1^2 t_2)\, dt_2\, dt_1 =$$

$$= -\int_{t_1=0}^{1}\int_{t_2=0}^{1} \left[20160\, t_1^5 t_2 + 18816\, t_1^2 t_2^3\right] dt_2\, dt_1 =$$

$$= -20160 \int_{t_1=0}^{1} t_1^5\, dt_1 \int_{t_2=0}^{1} t_2\, dt_2 - 18816 \int_{t_1=0}^{1} t_1^2\, dt_1 \cdot \int_{t_2=0}^{1} t_2^3\, dt_2 =$$

$$= -20160\cdot\frac{1}{6}\cdot\frac{1}{2} - 18816\cdot\frac{1}{3}\cdot\frac{1}{4} =$$

$$= -1680 - 1568 = \underline{\underline{-3248}}$$

HF) $S_0 := Pol_3(\mathbb{R}) = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 : a_0, \dots, a_3 \in \mathbb{R}\}$

$X = \begin{bmatrix} x_0 \\ \vdots \\ x_3 \end{bmatrix}$ $\quad x_i(p)\big|_{p=a_0+\dots+a_3x^3} = a_i$

X 4 dim koord S_0-on

Egyébi feladat az X koord. rsz-rel