```
>   with(linalg):
>   print('QR_FELBONTAS');
>   print('_____');
>   print('PYTHAGORASI_HARMASOK');
>   (u^2-v^2)^2+(2*u*v)^2=(u^2+v^2)^2;
>   #simplify((u^2-v^2)^2+(2*u*v)^2-(u^2+v^2)^2);
>   for v from 1 to 3 do:
>   for u from v+1 to 4 do:
>   print(u^2-v^2,2*u*v,u^2+v^2,'___',min((u^2-v^2)/(u^2+v^2),2*u*v/(u^2+v
>   ^2)),max((u^2-v^2)/(u^2+v^2),2*u*v/(u^2+v^2))):
>   od:od:
>   a1:=3/5; b1:=4/5; a2:=8/17; b2:=15/17; a3:=5/13; b3:=12/13; a4:=7/25;
>   b4:=24/25;
```

$$QR\_FELBONTAS$$

$$--$$

$$PYTHAGORASI\_HARMASOK$$

$$1681 = 1681$$

$$3,\ 4,\ 5,\ \text{---},\ \frac{3}{5},\ \frac{4}{5}$$

$$8,\ 6,\ 10,\ \text{---},\ \frac{3}{5},\ \frac{4}{5}$$

$$15,\ 8,\ 17,\ \text{---},\ \frac{8}{17},\ \frac{15}{17}$$

$$5,\ 12,\ 13,\ \text{---},\ \frac{5}{13},\ \frac{12}{13}$$

$$12,\ 16,\ 20,\ \text{---},\ \frac{3}{5},\ \frac{4}{5}$$

$$7, 24, 25, \text{---}, \frac{7}{25}, \frac{24}{25}$$

$$a1 := \frac{3}{5}$$

$$b1 := \frac{4}{5}$$

$$a2 := \frac{8}{17}$$

$$b2 := \frac{15}{17}$$

$$a3 := \frac{5}{13}$$

$$b3 := \frac{12}{13}$$

$$a4 := \frac{7}{25}$$

$$b4 := \frac{24}{25}$$

```
>  print('Feladat_visszafele');
>  R:=matrix(3,3,[2,1,1,0,-3,4,0,0,5]);
>  T12:=matrix(3,3,[a1,-b1,0,b1,a1,0,0,0,1]);
>  T13:=matrix(3,3,[a4,0,b4,0,1,0,-b4,0,a4]);
>  T23:=matrix(3,3,[1,0,0,0,b4,a4,0,a4,-b4]);
>  A:=evalm(3125*transpose(T12)&*transpose(T13)&*transpose(T23)&*R);
>  print(T12_,A_,'____',evalm(T12&*A));
```

Feladat_visszafele

$$R := \begin{bmatrix} 2 & 1 & 1 \\ 0 & -3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$T12 := \begin{bmatrix} \dfrac{3}{5} & \dfrac{-4}{5} & 0 \\[2mm] \dfrac{4}{5} & \dfrac{3}{5} & 0 \\[2mm] 0 & 0 & 1 \end{bmatrix}$$

$$T13 := \begin{bmatrix} \dfrac{7}{25} & 0 & \dfrac{24}{25} \\[2mm] 0 & 1 & 0 \\[2mm] \dfrac{-24}{25} & 0 & \dfrac{7}{25} \end{bmatrix}$$

$$T23 := \begin{bmatrix} 1 & 0 & 0 \\[2mm] 0 & \dfrac{24}{25} & \dfrac{7}{25} \\[2mm] 0 & \dfrac{7}{25} & \dfrac{-24}{25} \end{bmatrix}$$

$$A := \begin{bmatrix} 1050 & -5163 & 20249 \\ -1400 & -8116 & 293 \\ 6000 & 2265 & -220 \end{bmatrix}$$

$$T12\_,\ A\_,\ \_\_\_\_,\ \begin{bmatrix} 1750 & 3395 & 11915 \\ 0 & -9000 & 16375 \\ 6000 & 2265 & -220 \end{bmatrix}$$

```
>  print('QR_felbontas');
>  print('A____',evalm(A));
>  print('T12____',T12,A,'_____',evalm(T12&*A));
>  print('T13____',T13,evalm(T12&*A),'_____',evalm(T13&*T12&*A));
>  print('T23____',T23,evalm(T13&*T12&*A),'_____',evalm(T23&*T13&*T12&
>  *A),'__R');
```

$$QR\_felbontas$$

$$A_{\_\_\_\_}, \begin{bmatrix} 1050 & -5163 & 20249 \\ -1400 & -8116 & 293 \\ 6000 & 2265 & -220 \end{bmatrix}$$

$$T12_{\_\_\_\_}, \begin{bmatrix} \dfrac{3}{5} & \dfrac{-4}{5} & 0 \\[2mm] \dfrac{4}{5} & \dfrac{3}{5} & 0 \\[2mm] 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1050 & -5163 & 20249 \\ -1400 & -8116 & 293 \\ 6000 & 2265 & -220 \end{bmatrix}, _____, \begin{bmatrix} 1750 & 3395 & 11915 \\ 0 & -9000 & 16375 \\ 6000 & 2265 & -220 \end{bmatrix}$$

$$T13_{\_\_\_\_}, \begin{bmatrix} \dfrac{7}{25} & 0 & \dfrac{24}{25} \\[2mm] 0 & 1 & 0 \\[2mm] \dfrac{-24}{25} & 0 & \dfrac{7}{25} \end{bmatrix}, \begin{bmatrix} 1750 & 3395 & 11915 \\ 0 & -9000 & 16375 \\ 6000 & 2265 & -220 \end{bmatrix}, _____, \begin{bmatrix} 6250 & 3125 & 3125 \\ 0 & -9000 & 16375 \\ 0 & -2625 & -11500 \end{bmatrix}$$

$$T23_{\_\_\_\_}, \begin{bmatrix} 1 & 0 & 0 \\[2mm] 0 & \dfrac{24}{25} & \dfrac{7}{25} \\[2mm] 0 & \dfrac{7}{25} & \dfrac{-24}{25} \end{bmatrix}, \begin{bmatrix} 6250 & 3125 & 3125 \\ 0 & -9000 & 16375 \\ 0 & -2625 & -11500 \end{bmatrix}, _____, \begin{bmatrix} 6250 & 3125 & 3125 \\ 0 & -9375 & 12500 \\ 0 & 0 & 15625 \end{bmatrix},$$

$$\_\_R$$

```
>  print(A=QR);
>  print(R=T23_T13_T12_A);
>  A=INV(T23_T13_T12)*R;
>  A=TRANSP(T23_T13_T12)*R;
>  Q=TRANSP(T23_T13_T12);
>  print('T13_T12___',evalm(T13&*T12));
>  print('T23_T13_T12___',evalm(T23&*T13&*T12));
>  print('Q___',evalm(transpose(T23&*T13&*T12)));
```

$$A = QR$$

$$R = T23\_T13\_T12\_A$$

$$A = \text{INV}(T23\_T13\_T12)\,R$$

$$A = \mathrm{TRANSP}(\mathit{T23\_T13\_T12})\,R$$

$$Q = \mathrm{TRANSP}(\mathit{T23\_T13\_T12})$$

$$\mathit{T13\_T12}\,{}_{\text{---}},\ \begin{bmatrix} \dfrac{21}{125} & \dfrac{-28}{125} & \dfrac{24}{25} \\[2mm] \dfrac{4}{5} & \dfrac{3}{5} & 0 \\[2mm] \dfrac{-72}{125} & \dfrac{96}{125} & \dfrac{7}{25} \end{bmatrix}$$

$$\mathit{T23\_T13\_T12}\,{}_{\text{---}},\ \begin{bmatrix} \dfrac{21}{125} & \dfrac{-28}{125} & \dfrac{24}{25} \\[2mm] \dfrac{1896}{3125} & \dfrac{2472}{3125} & \dfrac{49}{625} \\[2mm] \dfrac{2428}{3125} & \dfrac{-1779}{3125} & \dfrac{-168}{625} \end{bmatrix}$$

$$Q\,{}_{\text{---}},\ \begin{bmatrix} \dfrac{21}{125} & \dfrac{1896}{3125} & \dfrac{2428}{3125} \\[2mm] \dfrac{-28}{125} & \dfrac{2472}{3125} & \dfrac{-1779}{3125} \\[2mm] \dfrac{24}{25} & \dfrac{49}{625} & \dfrac{-168}{625} \end{bmatrix}$$

```
>   restart:  with(linalg):
>   print('_____');
>   print('FOTENGELY_TRANSZFORMACIO');
>   print('_____');
>   print('Feladat_elokeszitese');
>   print('Szimm_3x3_matrix');
>   A=Q*Lambda*TRANP(Q);
>   Lambda:=diag(0,sqrt(2),-sqrt(2)):
>   Q:=matrix(3,3,[3/5,0,4/5,0,1,0,-4/5,0,3/5]):
>   print(Lambda=evalm(Lambda),'____',Q=evalm(Q));
>   print(A=evalm(Q)&*evalm(Lambda)&*evalm(transpose(Q)));
>   A:=evalm(Q&*Lambda&*transpose(Q));
>   print('SAJATVEKTOROK_Q_OSZLOPAI___',matrix(3,1,[Q[1,1],Q[2,1],Q[3,1]])
>   ,
>   matrix(3,1,[Q[1,2],Q[2,2],Q[3,2]]),matrix(3,1,[Q[1,3],Q[2,3],Q[3,3]]))
>   ;
>   print('CHAPRPOLY___',(lambda-Lambda[1,1])*(lambda-Lambda[2,2])*(lambda
>   -Lambda[3,3])=
>   collect((lambda)*(lambda-sqrt(2))*(lambda+sqrt(2)),lambda),'___',DET(A
>   -lambda*Id));

Warning, new definition for norm

Warning, new definition for trace
```

_____

FOTENGELY_TRANSZFORMACIO

_____

Feladat_elokeszitese

Szimm_3x3_matrix

$$A = Q\,\Lambda\,\mathrm{TRANP}(Q)$$

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix}, \text{----}, \ Q = \begin{bmatrix} \dfrac{3}{5} & 0 & \dfrac{4}{5} \\[2mm] 0 & 1 & 0 \\[2mm] \dfrac{-4}{5} & 0 & \dfrac{3}{5} \end{bmatrix}$$

$$A = \left(\begin{bmatrix} \dfrac{3}{5} & 0 & \dfrac{4}{5} \\[2mm] 0 & 1 & 0 \\[2mm] \dfrac{-4}{5} & 0 & \dfrac{3}{5} \end{bmatrix} \&* \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix} \right) \&* \begin{bmatrix} \dfrac{3}{5} & 0 & \dfrac{-4}{5} \\[2mm] 0 & 1 & 0 \\[2mm] \dfrac{4}{5} & 0 & \dfrac{3}{5} \end{bmatrix}$$

$$A := \begin{bmatrix} -\dfrac{16}{25}\sqrt{2} & 0 & -\dfrac{12}{25}\sqrt{2} \\[2mm] 0 & \sqrt{2} & 0 \\[2mm] -\dfrac{12}{25}\sqrt{2} & 0 & -\dfrac{9}{25}\sqrt{2} \end{bmatrix}$$

$$SAJATVEKTOROK\_Q\_OSZLOPAI\text{---}, \ \begin{bmatrix} \dfrac{3}{5} \\[2mm] 0 \\[2mm] \dfrac{-4}{5} \end{bmatrix}, \ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ \begin{bmatrix} \dfrac{4}{5} \\[2mm] 0 \\[2mm] \dfrac{3}{5} \end{bmatrix}$$

$$CHAPRPOLY\text{---}, \ \lambda\,(\lambda - \sqrt{2})\,(\lambda + \sqrt{2}) = \lambda^3 - 2\,\lambda, \ \text{---}, \ \mathrm{DET}(A - \lambda\,Id)$$

```
>   print('Feladat__ADOTT_A_____KERESENDO_Q_Lambda');
>   A := matrix([[-16/25*sqrt(2), 0, -12/25*sqrt(2)], [0, sqrt(2),
0],
>   [-12/25*sqrt(2), 0, -9/25*sqrt(2)]]);
>   print('SAJATERTEKEK___Lambda_ATLOJABAN____',CHARPOLY(A),'GYOKEI');
>   print(P=DET(A-lambda*Id),'____DET',evalm(A-lambda*diag(1,1,1)));
>   P:=evalm(det(A-lambda*diag(1,1,1)));
>   P:=factor(P);
>   P:=lambda*(lambda-sqrt(2))*(lambda+sqrt(2));
>   Lambda:=diag(0,sqrt(2),-sqrt(2));
>   print('SAJATERTEKEK__',0,sqrt(2),-sqrt(2));
>   print('SAJATVEKTOR__',0,'__SAJATERTEKHEZ_____',A*q1=0*q1);
>   q1:=matrix(3,1):  q2:=matrix(3,1):  q3:=matrix(3,1):
>   evalm(A)&*evalm(q1)=0;
>   evalm(A&*q1)=0;
>   print('MINDIG_EGGYEL_KEVESEBB_FUGGETLEN_EGYENLET___PLUSZ_',q[1,1]^2+q[
>   2,1]^2+q[3,1]^2=1);
>   print(-16/25*sqrt(2)*q1[1,1]-12/25*sqrt(2)*q1[3,1]=0,'___',sqrt(2)*q1[
>   2,1]=0);
>   solve(
>   {-16/25*sqrt(2)*q1[1,1]-12/25*sqrt(2)*q1[3,1]=0,sqrt(2)*q1[2,1]=0,
>   q1[1,1]^2+q1[2,1]^2+q1[3,1]^2=1},{q1[1,1],q1[2,1],q1[3,1]});
>   print('VALASZTAS__',q1[3,1]=3/5,'____',q1=matrix(3,1,[3/5,0,-4/5]),'__
>   ____',
>   Q=matrix(3,3,[3/5,x,x,0,x,x,-4/5,x,x]));
>   print('___');
>   print('SAJATVEKTOR__',sqrt(2),'__SAJATERTEKHEZ_____',A*q2=sqrt(2)*q2)
>   ;
>   evalm(A)&*evalm(q2)=evalm(sqrt(2)*q2);
>   evalm(A&*q2)=evalm(sqrt(2)*q2);
>   print('KETTO_FUGGETLEN_EGYENLET');
>   print(-16/25*sqrt(2)*q2[1,1]-12/25*sqrt(2)*q2[3,1]=sqrt(2)*q2[1,1],'__
>   _',
>   -12/25*sqrt(2)*q2[1,1]-9/25*sqrt(2)*q2[3,1]=sqrt(2)*q2[3,1]);
>   solve({-16/25*sqrt(2)*q2[1,1]-12/25*sqrt(2)*q2[3,1]=sqrt(2)*q2[1,1],
>   -12/25*sqrt(2)*q2[1,1]-9/25*sqrt(2)*q2[3,1]=sqrt(2)*q2[3,1],
>   q2[1,1]^2+q2[2,1]^2+q2[3,1]^2=1},{q2[1,1],q2[2,1],q2[3,1]});
>   print('VALASZTAS__',q2[2,1]=1,'____',q2=matrix(3,1,[0,1,0]),'_____',
>   Q=matrix(3,3,[3/5,0,x,0,1,x,-4/5,0,x]));
>   print('_____');
>   print('HAZI_FELADAT_____SAJATVEKTOR__',-sqrt(2),'__SAJATERTEKHEZ');
```

Feladat__ADOTT_A_____KERESENDO_Q_Lambda

$$A := \begin{bmatrix} -\dfrac{16}{25}\sqrt{2} & 0 & -\dfrac{12}{25}\sqrt{2} \\ 0 & \sqrt{2} & 0 \\ -\dfrac{12}{25}\sqrt{2} & 0 & -\dfrac{9}{25}\sqrt{2} \end{bmatrix}$$

$SAJATERTEKEK\ _{---}Lambda\ _ATLOJABAN\ _{----}$, CHARPOLY$(A)$, $GYOKEI$

$$\lambda\,(\lambda-\sqrt{2})\,(\lambda+\sqrt{2}) = \mathrm{DET}(A-\lambda\,Id),\ _{----}DET, \begin{bmatrix} -\dfrac{16}{25}\sqrt{2}-\lambda & 0 & -\dfrac{12}{25}\sqrt{2} \\[2mm] 0 & \sqrt{2}-\lambda & 0 \\[2mm] -\dfrac{12}{25}\sqrt{2} & 0 & -\dfrac{9}{25}\sqrt{2}-\lambda \end{bmatrix}$$

$$P := 2\,\lambda - \lambda^3$$

$$P := -\lambda\,(-2+\lambda^2)$$

$$P := \lambda\,(\lambda-\sqrt{2})\,(\lambda+\sqrt{2})$$

$$\Lambda := \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix}$$

$SAJATERTEKEK\ _{--}, 0,\ \sqrt{2},\ -\sqrt{2}$

$SAJATVEKTOR\ _{--}, 0,\ _{--}SAJATERTEKHEZ\ _{------}, A\,q1 = 0$

$$\begin{bmatrix} -\dfrac{16}{25}\sqrt{2} & 0 & -\dfrac{12}{25}\sqrt{2} \\[2mm] 0 & \sqrt{2} & 0 \\[2mm] -\dfrac{12}{25}\sqrt{2} & 0 & -\dfrac{9}{25}\sqrt{2} \end{bmatrix} \&* \begin{bmatrix} q1_{1,1} \\ q1_{2,1} \\ q1_{3,1} \end{bmatrix} = 0$$

$$\begin{bmatrix} -\dfrac{16}{25}\sqrt{2}\,q1_{1,1} - \dfrac{12}{25}\sqrt{2}\,q1_{3,1} \\[2mm] \sqrt{2}\,q1_{2,1} \\[2mm] -\dfrac{12}{25}\sqrt{2}\,q1_{1,1} - \dfrac{9}{25}\sqrt{2}\,q1_{3,1} \end{bmatrix} = 0$$

MINDIG_EGGYEL_KEVESEBB_FUGGETLEN_EGYENLET___PLUSZ_,

$$q_{1,1}{}^2 + q_{2,1}{}^2 + q_{3,1}{}^2 = 1$$

$$-\frac{16}{25}\sqrt{2}\,q1_{1,1} - \frac{12}{25}\sqrt{2}\,q1_{3,1} = 0, \ \text{---}, \ \sqrt{2}\,q1_{2,1} = 0$$

$$\{q1_{2,1} = 0,\ q1_{3,1} = \frac{-4}{5},\ q1_{1,1} = \frac{3}{5}\}, \ \{q1_{2,1} = 0,\ q1_{1,1} = \frac{-3}{5},\ q1_{3,1} = \frac{4}{5}\}$$

VALASZTAS$_{\text{--}}$, $q1_{3,1} = \dfrac{3}{5}$, ----, $q1 = \begin{bmatrix} \dfrac{3}{5} \\[2mm] 0 \\[2mm] \dfrac{-4}{5} \end{bmatrix}$, ------, $Q = \begin{bmatrix} \dfrac{3}{5} & x & x \\[2mm] 0 & x & x \\[2mm] \dfrac{-4}{5} & x & x \end{bmatrix}$

SAJATVEKTOR$_{\text{--}}$, $\sqrt{2}$, $_{\text{--}}$*SAJATERTEKHEZ*$_{\text{------}}$, $A\,q2 = \sqrt{2}\,q2$

$$\begin{bmatrix} -\dfrac{16}{25}\sqrt{2} & 0 & -\dfrac{12}{25}\sqrt{2} \\[2mm] 0 & \sqrt{2} & 0 \\[2mm] -\dfrac{12}{25}\sqrt{2} & 0 & -\dfrac{9}{25}\sqrt{2} \end{bmatrix} \&* \begin{bmatrix} q2_{1,1} \\ q2_{2,1} \\ q2_{3,1} \end{bmatrix} = \begin{bmatrix} \sqrt{2}\,q2_{1,1} \\ \sqrt{2}\,q2_{2,1} \\ \sqrt{2}\,q2_{3,1} \end{bmatrix}$$

$$\left[\begin{array}{c} -\dfrac{16}{25}\sqrt{2}\,q2_{1,1} - \dfrac{12}{25}\sqrt{2}\,q2_{3,1} \\[2mm] \sqrt{2}\,q2_{2,1} \\[2mm] -\dfrac{12}{25}\sqrt{2}\,q2_{1,1} - \dfrac{9}{25}\sqrt{2}\,q2_{3,1} \end{array}\right] = \left[\begin{array}{c} \sqrt{2}\,q2_{1,1} \\[2mm] \sqrt{2}\,q2_{2,1} \\[2mm] \sqrt{2}\,q2_{3,1} \end{array}\right]$$

$$KETTO\_FUGGETLEN\_EGYENLET$$

$$-\frac{16}{25}\sqrt{2}\,q2_{1,1} - \frac{12}{25}\sqrt{2}\,q2_{3,1} = \sqrt{2}\,q2_{1,1},\ \ ---,\ \ -\frac{12}{25}\sqrt{2}\,q2_{1,1} - \frac{9}{25}\sqrt{2}\,q2_{3,1} = \sqrt{2}\,q2_{3,1}$$

$$\{q2_{3,1} = 0,\ q2_{1,1} = 0,\ q2_{2,1} = 1\},\ \{q2_{3,1} = 0,\ q2_{1,1} = 0,\ q2_{2,1} = -1\}$$

$$VALASZTAS_{--},\ q2_{2,1} = 1,\ ----,\ q2 = \left[\begin{array}{c} 0 \\ 1 \\ 0 \end{array}\right],\ ------,\ Q = \left[\begin{array}{ccc} \dfrac{3}{5} & 0 & x \\[2mm] 0 & 1 & x \\[2mm] \dfrac{-4}{5} & 0 & x \end{array}\right]$$

$$-------$$

$$HAZI\_FELADAT_____SAJATVEKTOR_{--},\ -\sqrt{2},\ _{--}SAJATERTEKHEZ$$

```
>  restart:  with(linalg):
Warning, new definition for norm
Warning, new definition for trace
```

```
>   print('_____');
>   print('SVD_FELBONTAS');
>   print('_____');
>   print(A=Q*Lambda*TRANSP(R));
>   print('Q_3x3_ORT_____R_2x2_ORT_____Lambda_POZ_DIAG_3x2_MATRIX');
>   print('Feladat_elokeszitese');
>   Q:=matrix(3,3,[7/25,0,-24/25,0,1,0,24/25,0,7/25]):
>   Lambda:=matrix(3,2,[3,0,0,2,0,0]):
>   R:=matrix(2,2,[3/5,4/5,-4/5,3/5]):
>   print(Q=evalm(Q),Lambda=evalm(Lambda),R=evalm(R));
>   A:=evalm(Q&*Lambda&*transpose(R)):
>   print(A=evalm(Q)&*evalm(Lambda)&*evalm(transpose(R)),'_____',evalm(
>   A));
```

$SVD\_FELBONTAS$

$$A = Q\,\Lambda\,\mathrm{TRANSP}(R)$$

$Q\_3x3\_ORT\_\_\_\_R\_2x2\_ORT\_\_\_\_\_Lambda\_POZ\_DIAG\_3x2\_MATRIX$

$Feladat\_elokeszitese$

$$Q = \begin{bmatrix} \dfrac{7}{25} & 0 & \dfrac{-24}{25} \\ 0 & 1 & 0 \\ \dfrac{24}{25} & 0 & \dfrac{7}{25} \end{bmatrix},\ \Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix},\ R = \begin{bmatrix} \dfrac{3}{5} & \dfrac{4}{5} \\ \dfrac{-4}{5} & \dfrac{3}{5} \end{bmatrix}$$

$$A = \left(\begin{bmatrix} \dfrac{7}{25} & 0 & \dfrac{-24}{25} \\ 0 & 1 & 0 \\ \dfrac{24}{25} & 0 & \dfrac{7}{25} \end{bmatrix} \&* \begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \right) \&* \begin{bmatrix} \dfrac{3}{5} & \dfrac{-4}{5} \\ \dfrac{4}{5} & \dfrac{3}{5} \end{bmatrix}, \text{--------}, \begin{bmatrix} \dfrac{63}{125} & \dfrac{-84}{125} \\ \dfrac{8}{5} & \dfrac{6}{5} \\ \dfrac{216}{125} & \dfrac{-288}{125} \end{bmatrix}$$

12

```
>    print('ADOTT__A_____KERESENDO_Q_Lambda_R');
>    A:=matrix([[63/125, -84/125], [8/5, 6/5], [216/125, -288/125]]);
>    transp(A)*A=transp(Q*Lambda*transp(R))*Q*Lambda*transp(R);
>    transp(A)*A=R*transp(Lambda)*transp(Q)*Q*Lambda*transp(R);
>    transp(A)*A=R*transp(Lambda)*Lambda*transp(R);
>    transp(A)*A=R*diag(lambda[1]^2,lambda[2]^2)*transp(R);
>    print('EZ__',B=transp(A)*A,'___FOTENGELY_ALAKJA');
>    B:=evalm(transpose(A)&*A):
>    print(B=evalm(transpose(A))&*evalm(A),'___',evalm(B));
>    print('B_SAJATERTEKEI___',DET(B-lambda*Id)=0);
>    CHARPOLY(B)=DET(evalm(B-lambda*diag(1,1)));
>    P:=evalm(det(B-lambda*diag(1,1)));
>    factor(P);
>    print(lambda[1]>=lambda[2],lambda[1]^2=9,lambda[2]^2=4);
>    print(lambda[1]=3,lambda[2]=2,'_____',Lambda=matrix(3,2,[3,0,0,2,0,0]
>    ));
>    print('R_OSZLOPAI_B_SAJATVEKTORAI___EGYSEGVEKTOROK');
>    print(R=[r[1],r[2]],'____',r[1],'_B_SAJATVEKT_3_SAJATERT___',r[2],'_B_
>    SAJATVEKT_2_SAJATERT');
>    print('HAZI_FELADAT____',r[1]=matrix(2,1,[3/5,-4/5]),'_PLUSZMINUSZ___'
>    ,r[2]=matrix(2,1,[3/5,-4/5]),'_PLUSZMINUSZ');
>    R=matrix([[3/5, 4/5], [-4/5, 3/5]]);
>    print(A*r[1]=Q*Lambda*transp(R)*r[1],'____',Q*lambda[1]*e[1],'____',la
>    mbda[1]*q[1]);
>    print(q[1]=(1/lambda[1])*A*r[1],'_____',q[2]=(1/lambda[2])*A*r[2]);
>    print('Q_ELSO_KET_OSZLOPA___',A*R*diag(lambda[1],lambda[2])^(-1),'____
>    ',
>    evalm(A)*evalm(R)*diag(1/lambda[1],1/lambda[2]));
>    Q=matrix(3,3,[7/25,0,x,0,1,x,24/25,0,x]);
>    print('A_HARMADIK_OSZLOP_TETSZOLEGES_',q[1],q[2],'_RE_MEROLEGES_EGYSEG
>    VEKTOR');
```

$$ADOTT\_\_A_____KERESENDO\_Q\_Lambda\_R$$

$$A := \begin{bmatrix} \dfrac{63}{125} & \dfrac{-84}{125} \\[2mm] \dfrac{8}{5} & \dfrac{6}{5} \\[2mm] \dfrac{216}{125} & \dfrac{-288}{125} \end{bmatrix}$$

$$\operatorname{transp}(A)\,A = \operatorname{transp}(Q\,\Lambda\,\operatorname{transp}(R))\,Q\,\Lambda\,\operatorname{transp}(R)$$

$$\operatorname{transp}(A)\,A = R\,\operatorname{transp}(\Lambda)\,\operatorname{transp}(Q)\,Q\,\Lambda\,\operatorname{transp}(R)$$

$$\operatorname{transp}(A)\,A = R\,\operatorname{transp}(\Lambda)\,\Lambda\,\operatorname{transp}(R)$$

$$\operatorname{transp}(A)\,A = R \begin{bmatrix} \lambda_1{}^2 & 0 \\ 0 & \lambda_2{}^2 \end{bmatrix} \operatorname{transp}(R)$$

$$EZ\_\_,\ B = \operatorname{transp}(A)\,A,\ \_\_\_FOTENGELY\_ALAKJA$$

$$B = \begin{bmatrix} \dfrac{63}{125} & \dfrac{8}{5} & \dfrac{216}{125} \\ \dfrac{-84}{125} & \dfrac{6}{5} & \dfrac{-288}{125} \end{bmatrix} \&* \begin{bmatrix} \dfrac{63}{125} & \dfrac{-84}{125} \\ \dfrac{8}{5} & \dfrac{6}{5} \\ \dfrac{216}{125} & \dfrac{-288}{125} \end{bmatrix},\ ---, \begin{bmatrix} \dfrac{29}{5} & \dfrac{-12}{5} \\ \dfrac{-12}{5} & \dfrac{36}{5} \end{bmatrix}$$

$$B\_SAJATERTEKEI\_\_\_,\ \mathrm{DET}(B - \lambda\,Id) = 0$$

$$\mathrm{CHARPOLY}(B) = \mathrm{DET}\left(\begin{bmatrix} \dfrac{29}{5} - \lambda & \dfrac{-12}{5} \\ \dfrac{-12}{5} & \dfrac{36}{5} - \lambda \end{bmatrix}\right)$$

$$P := 36 - 13\,\lambda + \lambda^2$$

$$(\lambda - 4)\,(\lambda - 9)$$

$$\lambda_2 \le \lambda_1,\ \lambda_1{}^2 = 9,\ \lambda_2{}^2 = 4$$

$$\lambda_1 = 3,\ \lambda_2 = 2,\ \text{------,}\ \Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$R\_OSZLOPAI\_B\_SAJATVEKTORAI\_\_\_EGYSEGVEKTOROK$$

$$R = [r_1, r_2], \text{----}, r_1, \_B\_SAJATVEKT\_3\_SAJATERT\_\_\_, r_2,$$
$$\_B\_SAJATVEKT\_2\_SAJATERT$$

$$HAZI\_FELADAT\text{----}, \; r_1 = \begin{bmatrix} \dfrac{3}{5} \\[2mm] \dfrac{-4}{5} \end{bmatrix}, \; \_PLUSZMINUSZ\text{---}, \; r_2 = \begin{bmatrix} \dfrac{3}{5} \\[2mm] \dfrac{-4}{5} \end{bmatrix}, \; \_PLUSZMINUSZ$$

$$R = \begin{bmatrix} \dfrac{3}{5} & \dfrac{4}{5} \\[2mm] \dfrac{-4}{5} & \dfrac{3}{5} \end{bmatrix}$$

$$A\,r_1 = Q\,\Lambda\,\mathrm{transp}(R)\,r_1, \; \text{----}, \; Q\,\lambda_1\,e_1, \; \text{----}, \; \lambda_1\,q_1$$

$$q_1 = \frac{A\,r_1}{\lambda_1}, \; \text{-----}, \; q_2 = \frac{A\,r_2}{\lambda_2}$$

$$Q\_ELSO\_KET\_OSZLOPA\_\_\_, \; \frac{A\,R}{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}, \; \text{----},$$

$$\begin{bmatrix} \dfrac{63}{125} & \dfrac{-84}{125} \\[2mm] \dfrac{8}{5} & \dfrac{6}{5} \\[2mm] \dfrac{216}{125} & \dfrac{-288}{125} \end{bmatrix} \begin{bmatrix} \dfrac{3}{5} & \dfrac{4}{5} \\[2mm] \dfrac{-4}{5} & \dfrac{3}{5} \end{bmatrix} \begin{bmatrix} \dfrac{1}{\lambda_1} & 0 \\[2mm] 0 & \dfrac{1}{\lambda_2} \end{bmatrix}$$

$$Q = \begin{bmatrix} \dfrac{7}{25} & 0 & x \\[2mm] 0 & 1 & x \\[2mm] \dfrac{24}{25} & 0 & x \end{bmatrix}$$

$A\_HARMADIK\_OSZLOP\_TETSZOLEGES\_$, q_1, q_2,
$\_RE\_MEROLEGES\_EGYSEGVEKTOR$

```
>   restart:  with(linalg):
>   print('_____',
>   );
>   print('ALTALANOS_INVERZ');
>   print('_____',
>   );
>   print('Elokeszites');
>   print('HARMADIK_SOR___2x_MASODIK_MINUS_ELSO');
>   A:=matrix(3,4,[2,4,8,16,1,2,7,15,0,0,6,14]);
>   print('QR_FELBONTAS_____',A=QR);
>   print(R=matrix(3,4,[a,b,c,d,0,e,f,g,0,0,0,0]),'_____',R0=matrix(2,
>   4,[a,b,c,d,0,e,f,g]));
>   print(Ainv=Rinv*Qinv):
>   print(Qinv=transp(Q),'____',Rinv=transp(R0)*inv(R0*transp(R0))*transp(
>   Q),'KIEGESZITVE_EGY_0_OSZLOPPAL');
>   print('_____');
>   A:=matrix(3,4,[2,4,8,16,1,2,7,15,0,0,6,14]);
>   print('QR_HEZ__T12__GIVENS_FORGATO_MATRIX');
>   T12:=matrix(3,3,[u,v,0,-v,u,0,0,0,1]);
>   u^2+v^2=1;
>   print(T12_*A_,'__MASODIK_OSZL_ELSO_ELEME_0');
>   T12_*A_ = evalm(T12&*A);
>   T12_*A_:=matrix([[x, x, x, x], [-2*v+u, x, x, x], [x, x, x, x]]);
>   print(-2*v+u=0,'__',u^2+v^2=1);
>   solve({-2*v+u=0,u^2+v^2=1},{u,v});
>   print('VALASZTAS____',u=1/sqrt(5),v=2/sqrt(5));
>   u:=2/sqrt(5):     v:=1/sqrt(5):
>   print(T12_*A_=evalm(T12)&*evalm(A),'___',evalm(T12&*A));
>   print('BAR_EZ_MAR_ROGTON_ALSO_TRIANG___TOVABB_ELIM');
>   T33:=matrix(3,3,[1,0,0,0,w,z,0,-z,w]);
>   w^2+z^2=1;
>   print(
>   T33_*T12_*A_=evalm(T33)&*matrix([[sqrt(5), 2*sqrt(5), 23/5*sqrt(5),
>   47/5*sqrt(5)], [0, 0, 6/5*sqrt(5), 14/5*sqrt(5)], [0, 0, 6, 14]]),
>   '____',matrix(3,4,[x,x,x,x,0,0,x,x,0,0,-6/sqrt(5)*z+6*w,-14/sqrt(5)*z+
>   14*w]));
>   print('KI_SEM_KELL_SZAMOLNI_w_z_ERTEKET____A_HARMADIK_SOR_0_LESZ');
>   print(-6/5*sqrt(5)*z+6*w=0,w^2+z^2=1);
>   solve({-6/5*sqrt(5)*z+6*w=0,w^2+z^2=1},{w,z});
>   print('VALASZTAS___',w=(1/sqrt(5))*sqrt(5/6),'__',z=sqrt(5/6));
>   w:=1/sqrt(6): z:=sqrt(5)/sqrt(6):
>   #print(w_=w,z_=z);
>   T33_=matrix(3,3,[1,0,0,0,1/sqrt(6),sqrt(5)/sqrt(6),0,-sqrt(5)/sqrt(6),
>   1/sqrt(6)]);
>   T_=T33_&*T12_,'___',evalm(T33&*T12);
>   print(Q_=Tinv_,'_',transp(T_),'_____',Qinv_=T_);
>   Qinv:=evalm(T33&*T12):
>   R_=evalm(T33&*T12&*A);
>   R0:=matrix(2,4,[sqrt(5), 2*sqrt(5), 23/5*sqrt(5), 47/5*sqrt(5),
0, 0,
>   6/5*sqrt(5)*sqrt(6), 14/5*sqrt(5)*sqrt(6)]);
>   R0inv_=transp(R0_)*inv(R0_*transp(R0_));
>   R0inv:=evalm(transpose(R0)&*inverse(R0&*transpose(R0)));
>   Rinv:=matrix([[29/765*sqrt(5), -199/9180*sqrt(5)*sqrt(6),0],
>   [58/765*sqrt(5), -199/4590*sqrt(5)*sqrt(6),0], [14/765*sqrt(5),
>   -113/18360*sqrt(5)*sqrt(6),0], [-2/255*sqrt(5),
>   89/6120*sqrt(5)*sqrt(6),0]]);
>   print(Ainv_=Rinv_*Qinv,'__',Rinv_*T_);
>   Ainv:=evalm(Rinv&*T33&*T12);
```

```
Warning, new definition for norm

Warning, new definition for trace
```

ALTALANOS_INVERZ

Elokeszites

HARMADIK_SOR___2x_MASODIK_MINUS_ELSO

$$A := \begin{bmatrix} 2 & 4 & 8 & 16 \\ 1 & 2 & 7 & 15 \\ 0 & 0 & 6 & 14 \end{bmatrix}$$

QR_FELBONTAS_____, A = QR

$$R = \begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{---------}, R0 = \begin{bmatrix} a & b & c & d \\ 0 & e & f & g \end{bmatrix}$$

Ainv = Rinv Qinv

$Qinv = \text{transp}(Q)$, ----, $Rinv = \text{transp}(R0)\,\text{inv}(R0\,\text{transp}(R0))\,\text{transp}(Q)$,
  KIEGESZITVE_EGY_0_OSZLOPPAL

$$A := \begin{bmatrix} 2 & 4 & 8 & 16 \\ 1 & 2 & 7 & 15 \\ 0 & 0 & 6 & 14 \end{bmatrix}$$

$QR\_HEZ\_\_T12\_\_GIVENS\_FORGATO\_MATRIX$

$$T12 := \begin{bmatrix} u & v & 0 \\ -v & u & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$u^2 + v^2 = 1$$

$T12\_A\_, \ \_\_MASODIK\_OSZL\_ELSO\_ELEME\_0$

$$T12\_A\_ = \begin{bmatrix} 2\,u+v & 4\,u+2\,v & 8\,u+7\,v & 16\,u+15\,v \\ -2\,v+u & -4\,v+2\,u & -8\,v+7\,u & -16\,v+15\,u \\ 0 & 0 & 6 & 14 \end{bmatrix}$$

$$T12\_A\_ = \begin{bmatrix} x & x & x & x \\ -2\,v+u & x & x & x \\ x & x & x & x \end{bmatrix}$$

$$-2\,v+u = 0, \ \_\_, \ u^2 + v^2 = 1$$

$$\{u = 2\,\text{RootOf}(5\,\_Z^2 - 1),\ v = \text{RootOf}(5\,\_Z^2 - 1)\}$$

$$VALASZTAS\_\_\_\_\_, \ u = \frac{1}{5}\,\sqrt{5}, \ v = \frac{2}{5}\,\sqrt{5}$$

19

$$T12\_A\_ = \begin{bmatrix} u & v & 0 \\ -v & u & 0 \\ 0 & 0 & 1 \end{bmatrix} \&* \begin{bmatrix} 2 & 4 & 8 & 16 \\ 1 & 2 & 7 & 15 \\ 0 & 0 & 6 & 14 \end{bmatrix}, \text{---}, \begin{bmatrix} \sqrt{5} & 2\sqrt{5} & \dfrac{23}{5}\sqrt{5} & \dfrac{47}{5}\sqrt{5} \\ 0 & 0 & \dfrac{6}{5}\sqrt{5} & \dfrac{14}{5}\sqrt{5} \\ 0 & 0 & 6 & 14 \end{bmatrix}$$

BAR_EZ_MAR_ROGTON_ALSO_TRIANG___TOVABB_ELIM

$$T33 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & w & z \\ 0 & -z & w \end{bmatrix}$$

$$w^2 + z^2 = 1$$

$$T33\_T12\_A\_ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & w & z \\ 0 & -z & w \end{bmatrix} \&* \begin{bmatrix} \sqrt{5} & 2\sqrt{5} & \dfrac{23}{5}\sqrt{5} & \dfrac{47}{5}\sqrt{5} \\ 0 & 0 & \dfrac{6}{5}\sqrt{5} & \dfrac{14}{5}\sqrt{5} \\ 0 & 0 & 6 & 14 \end{bmatrix}, \text{----},$$

$$\begin{bmatrix} x & x & x & x \\ 0 & 0 & x & x \\ 0 & 0 & -\dfrac{6}{5}\sqrt{5}\,z + 6\,w & -\dfrac{14}{5}\sqrt{5}\,z + 14\,w \end{bmatrix}$$

KI_SEM_KELL_SZAMOLNI_w_z_ERTEKET____A_HARMADIK_SOR_0_LESZ

$$-\frac{6}{5}\sqrt{5}\,z + 6\,w = 0,\; w^2 + z^2 = 1$$

$$\{z = \text{RootOf}(6\,\_Z^2 - 5),\; w = \frac{1}{5}\sqrt{5}\,\text{RootOf}(6\,\_Z^2 - 5)\}$$

$$VALASZTAS\_\_\_,\; w = \frac{1}{30}\sqrt{5}\,\sqrt{30},\; \_\_,\; z = \frac{1}{6}\sqrt{30}$$

$$T33_- = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1}{6}\sqrt{6} & \dfrac{1}{6}\sqrt{5}\sqrt{6} \\ 0 & -\dfrac{1}{6}\sqrt{5}\sqrt{6} & \dfrac{1}{6}\sqrt{6} \end{bmatrix}$$

$$T_- = T33_- \,\&*\, T12_-, \; \text{---}, \begin{bmatrix} \dfrac{2}{5}\sqrt{5} & \dfrac{1}{5}\sqrt{5} & 0 \\ -\dfrac{1}{30}\sqrt{5}\sqrt{6} & \dfrac{1}{15}\sqrt{5}\sqrt{6} & \dfrac{1}{6}\sqrt{5}\sqrt{6} \\ \dfrac{1}{6}\sqrt{6} & -\dfrac{1}{3}\sqrt{6} & \dfrac{1}{6}\sqrt{6} \end{bmatrix}$$

$$Q_- = Tinv_-, \; \_, \; \mathrm{transp}(T_-), \; \text{--------}, \; Qinv_- = T_-$$

$$R_- = \begin{bmatrix} \sqrt{5} & 2\sqrt{5} & \dfrac{23}{5}\sqrt{5} & \dfrac{47}{5}\sqrt{5} \\ 0 & 0 & \dfrac{6}{5}\sqrt{5}\sqrt{6} & \dfrac{14}{5}\sqrt{5}\sqrt{6} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R0 := \begin{bmatrix} \sqrt{5} & 2\sqrt{5} & \dfrac{23}{5}\sqrt{5} & \dfrac{47}{5}\sqrt{5} \\ 0 & 0 & \dfrac{6}{5}\sqrt{5}\sqrt{6} & \dfrac{14}{5}\sqrt{5}\sqrt{6} \end{bmatrix}$$

$$R0inv_- = \mathrm{transp}(R0_-)\,\mathrm{inv}(R0_-\,\mathrm{transp}(R0_-))$$

$$R0inv := \begin{bmatrix} \dfrac{29}{765}\sqrt{5} & -\dfrac{199}{9180}\sqrt{5}\sqrt{6} \\ \dfrac{58}{765}\sqrt{5} & -\dfrac{199}{4590}\sqrt{5}\sqrt{6} \\ \dfrac{14}{765}\sqrt{5} & -\dfrac{113}{18360}\sqrt{5}\sqrt{6} \\ -\dfrac{2}{255}\sqrt{5} & \dfrac{89}{6120}\sqrt{5}\sqrt{6} \end{bmatrix}$$

$$Rinv := \begin{bmatrix} \dfrac{29}{765}\,\sqrt{5} & -\dfrac{199}{9180}\,\sqrt{5}\,\sqrt{6} & 0 \\[2ex] \dfrac{58}{765}\,\sqrt{5} & -\dfrac{199}{4590}\,\sqrt{5}\,\sqrt{6} & 0 \\[2ex] \dfrac{14}{765}\,\sqrt{5} & -\dfrac{113}{18360}\,\sqrt{5}\,\sqrt{6} & 0 \\[2ex] -\dfrac{2}{255}\,\sqrt{5} & \dfrac{89}{6120}\,\sqrt{5}\,\sqrt{6} & 0 \end{bmatrix}$$

$$Ainv_- = Rinv_-\,Qinv,\; \_\_,\; Rinv_-\,T_-$$

$$Ainv := \begin{bmatrix} \dfrac{179}{1836} & \dfrac{-5}{918} & \dfrac{-199}{1836} \\[2ex] \dfrac{179}{918} & \dfrac{-5}{459} & \dfrac{-199}{918} \\[2ex] \dfrac{157}{3672} & \dfrac{11}{1836} & \dfrac{-113}{3672} \\[2ex] \dfrac{-37}{1224} & \dfrac{13}{612} & \dfrac{89}{1224} \end{bmatrix}$$