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On prime JB*-triples. (English)

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A complex Banach space A together with a continuous triple product $A^3 \ni (a,b,c) \mapsto \{abc\} \in A$ is called a JB*-triple if it satisfies the following conditions (i)–(iv): (i) $\{abc\}$ is symmetric and bilinear in a, c and conjugate linear in b; (ii) $\{xy\{abc\}\} = \{\{xya\}bc\} + \{ab\{xyc\}\} - \{a\{yxb\}c\}$; (iii) the operator $x \mapsto \{aax\}$ is Hermitian with positive spectrum; (iv) $\|\{aaa\}\| = \|a\|^3$. Every C^*-algebra is a JB*-triple via $\{abc\} = \frac{1}{2}(ab^*c + cb^*a)$. More generally, every JB*-algebra with Jordan product $(a,b) \mapsto a \circ b$ is a JB*-triple with respect to $\{abc\} = (a \circ b^*) \circ c + (b^* \circ c) \circ a - (a \circ c) \circ b^*$. A JB*-triple isometric to a subtriple of a C^*-algebra is called a JC*-triple. A JB*-triple A is said to be prime if for $x, y \in A, Q_{x,y} = 0$ implies $x = 0$ or $y = 0$, where $Q_{a,b}$ is defined as $Q_{a,b}(x) = \{axb\}$.
This paper is devoted to results concerning the existence of a universal constant $K > 0$ such that for any prime JB*-triple A and $a, b \in A$ we have $\|Q_{a,b}\| \geq K\|a\| \cdot \|b\|$. For prime JB*-algebras representable on a complex Hilbert space, known as JC*-algebras, an admissible value of $K = \frac{1}{20\,412}$ was given for the universal constant, and the problem for a prime exceptional JB*-algebra was left open. The purpose of the present paper is both to sharpen and to extend this result for any prime JB*-triples.
The main result of the paper is Theorem 4.3: Let A be a prime JB*-triple, and let $a, b \in A$. Then $\|Q_{a,b}\| \geq \frac{1}{6}\|a\| \cdot \|b\|$. Further, if A is (i) a JC*-triple, then $\|Q_{a,b}\| \geq \frac{1}{4}\|a\| \cdot \|b\|$; (ii) a C^*-algebra, then $\|Q_{a,b}\| \geq (\sqrt{2} - 1)\|a\| \cdot \|b\|$.

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