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Generalization of an inequality of G. Pólya concerning the eigenfrequencies of vibrating bodies. (English)

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The authors treat the behavior of the eigenvalue of the Dirichlet problem

$$\Delta u + \Lambda^2 u(x) = 0, \quad x \in \Omega, \quad u|_{\partial\Omega} = 0$$

on a domain $\Omega \subset R^N$, whose closure is C^2-diffeomorphic with the closure of the open unit ball B^N in R^N. They say that the result of Pólya

$$\sup_{\Omega\in\mathcal{C}} [\Lambda_1(\Omega)\frac{area\Omega}{length\partial\Omega}] = \frac{\pi}{2},$$

where $\mathcal{C}$ denotes the family of all open convex subsets of R^2, can be applied for N-dimensional ($N \geq 3$) convex bodies too. Such a class K can be found in the following theorem: Denote by K the set of those domains $\Omega \subset R^N (N \geq 3)$ for which there exists a C^2-diffeomorphism $T : B^N \leftrightarrow \Omega$ and for which the Minkowski curvature with respect to the normal vector of $\partial\Omega$ (directed outward) is non-negative at any point of $\partial\Omega$. Then

$$c_1(K) = \sup_{\Omega\in K} [\Lambda_1(\Omega)\frac{vol_N\Omega}{vol_{N-1}\partial\Omega}] = \frac{\pi}{2}.$$

A proof of the theorem, using three lemmas, is given. Under the hypothesis of the theorem one can get, using the method developed in the paper, the estimate

$$R_j(\Omega) \leq (j - \frac{1}{2})\pi, \quad j = 1, 2, ..., \quad \Omega \in K,$$

where

$$R_j(\Omega) = \Lambda_j(\Omega)\frac{vol_N\Omega}{vol_{N-1}\partial\Omega}.$$

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Keywords : eigenfrequencies of vibrating bodies; Pólya inequality; Dirichlet problem
Classification :
∗49R50 Variational methods for eigenvalues of operators
35J05 Laplace equation, etc.
35P15 Estimation of eigenvalues for PD operators
58J50 Spectral problems; spectral geometry; scattering theory
52A20 Convex sets in n dimensions
74H45 Vibrations
28A75 Geometric measure theory
49Q15 Geometric measure and integration theory, etc.