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## The fitting and functional analysis of a double rotor potential energy surface for the *R* and *S* enantiomers of 1-chloro-3-fluoro-isobutane

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### Abstract

A model compound was chosen to see whether it mimics a backbone of an amino acid residue in a peptide structure so that a model Ramachandran potential energy surface could be fitted by a mathematical function. A Fourier series of two independent variables ( $\varphi$  and  $\psi$ ) has been used to fit a set of grid points representing the surface. To determine the accuracy of the fitted equation vs. the generated data points three grids were examined,  $24^2 = 576$  points ( $15^\circ$  intervals),  $12^2 = 144$  points ( $30^\circ$  intervals), and  $6^2 = 36$  points ( $60^\circ$  intervals). The grid points were generated for the *S* enantiomer and a Fourier expansion was fitted to the grid points along with a functional analysis of each fitted expansion. A series of functions were found for 15, 30, and  $60^\circ$  increments in order to see the lowest limit of resolution of the grid needed for a relatively accurate fit. Ab initio calculations were also carried out for the *R* and *S* enantiomer to fit a 31 term Fourier expansion where a functional analysis determined the location of the critical points from the expansions. Geometry optimizations were performed to locate more precisely the minima. The optimized minima were then included in a new surface that was fit.

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**Keywords:** Fourier expansion; Functional analysis; Geometry optimization; Minimum energy conformer; Potential energy surface; Double rotor

### 1. Introduction

A great impact in medicine will arise when one can determine the rules of protein folding. Since a protein is made up of a sequence of amino acids, one way of attempting to understand the folding process is to individually look at each amino acid residue and locate the potential energy minima of each structure.

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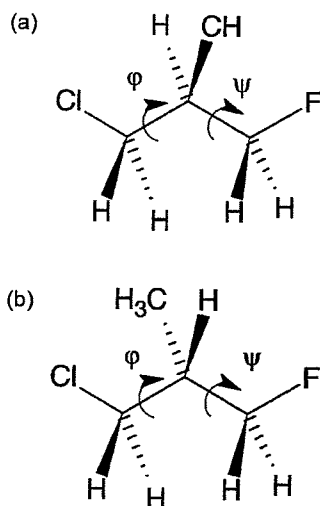


Fig. 1. The fully extended conformation of 1-chloro-3-fluoroisobutane. (a) *S* absolute configuration. (b) *R* absolute configuration.

The conformation where the potential energy is a minimum would be the approximate conformation each residue would be found in the protein. A gradual process to find the location of potential energy minima would then be performed to di-peptides, tri-peptides, and so on, to see how each residue is affected by its neighbour. Several questions may be asked. What about the relationships between all of these surfaces? Is there some sort of natural relationship between these surfaces that can be found?

Mathematics has been used for centuries to describe the known and to predict the unknown. Fourier expansions can fit accurately conformational potential energy surfaces and hyper surfaces which describe the surface as a linear combination of trigonometric functions rather than a set of generated data points. This expansion summarizes the surface to a great accuracy, where the location of the minima can be simply found by a functional analysis.

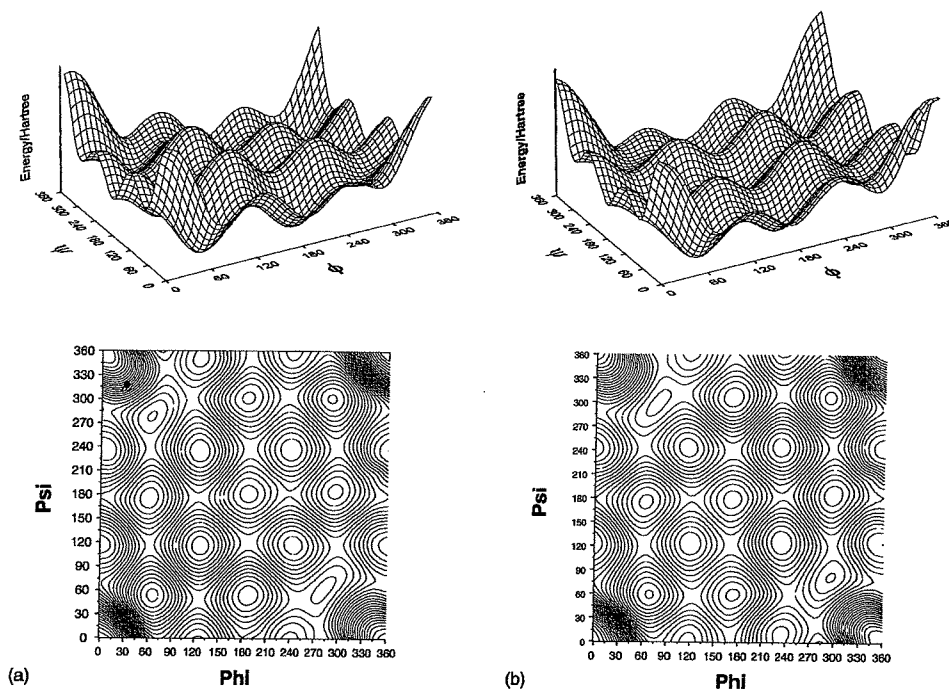


Fig. 2. (a) Ramachandran type PEHS of the *S* configuration model molecule. (Top) Landscape representation. (Bottom) Contour diagram representation. (b) Ramachandran type PEHS of the *R* configuration model molecule. (Top) Landscape representation. (Bottom) Contour diagram representation.

Table 1  
Tabulated components of the fitted 28 term Fourier expansion for S-1-chloro-3-fluoro-isobutane

Ar 15°							
-461.30844948	0.64809403 sin $\psi$	-0.56742157 sin 2 $\psi$	-0.69022701 sin 3 $\psi$	1.32820141 cos $\psi$	0.35652097 cos 2 $\psi$	4.16135608 cos 3 $\psi$	
1.21185436 sin $\varphi$	-2.49021582 sin $\psi$ sin $\varphi$	-1.40061795 sin 2 $\psi$ sin $\varphi$	-1.14850532 sin 3 $\psi$ sin $\varphi$	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	0 cos 3 $\psi$ sin $\varphi$	
-0.87790882 sin 2 $\varphi$	-1.44918333 sin $\psi$ sin 2 $\varphi$	-1.25231971 sin 2 $\psi$ sin 2 $\varphi$	-0.82453399 sin 3 $\psi$ sin 2 $\varphi$	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	0 cos 3 $\psi$ sin 2 $\varphi$	
0.64715046 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	0 sin 2 $\psi$ sin 3 $\varphi$	-0.64472843 sin 3 $\psi$ sin 3 $\varphi$	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	0 cos 3 $\psi$ sin 3 $\varphi$	
2.57596399 cos $\varphi$	0 sin $\psi$ cos $\varphi$	0 sin 2 $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$	4.41334861 cos $\psi$ cos $\varphi$	1.80832742 cos 2 $\psi$ cos $\varphi$	0.53858137 cos 3 $\psi$ cos $\varphi$	
1.74543674 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 2 $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$	1.63413861 cos $\psi$ cos 2 $\varphi$	1.25253585 cos 2 $\psi$ cos 2 $\varphi$	0.65948369 cos 3 $\psi$ cos 2 $\varphi$	
4.52198337 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	0 sin 2 $\psi$ cos 3 $\varphi$	0 sin 3 $\psi$ cos 3 $\varphi$	0.25352051 cos $\psi$ cos 3 $\varphi$	0 cos 2 $\psi$ cos 3 $\varphi$	0.47723389 cos 3 $\psi$ cos 3 $\varphi$	
Ar 30°							
-461.30986944	0.65080338 sin $\psi$	-0.56700006 sin 2 $\psi$	-0.68923750 sin 3 $\psi$	1.32753090 cos $\psi$	0.36438819 cos 2 $\psi$	4.16949583 cos 3 $\psi$	
1.20991044 sin $\varphi$	-2.49149438 sin $\psi$ sin $\varphi$	-1.39762635 sin 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	0 cos 3 $\psi$ sin $\varphi$	
-0.87788755 sin 2 $\varphi$	-1.45250963 sin $\psi$ sin 2 $\varphi$	-1.25563542 sin 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	0 cos 3 $\psi$ sin 2 $\varphi$	
0.64848333 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	0 sin 2 $\psi$ sin 3 $\varphi$	0 sin 3 $\psi$ sin 3 $\varphi$	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	0 cos 3 $\psi$ sin 3 $\varphi$	
2.57571291 cos $\varphi$	0 sin $\psi$ cos $\varphi$	0 sin 2 $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$	4.41330725 cos $\psi$ cos $\varphi$	1.80465439 cos 2 $\psi$ cos $\varphi$	0.53186654 cos 3 $\psi$ cos $\varphi$	
1.74622222 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 2 $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$	1.62978494 cos $\psi$ cos 2 $\varphi$	1.24906458 cos 2 $\psi$ cos 2 $\varphi$	0.70300000 cos 3 $\psi$ cos 2 $\varphi$	
4.52506944 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	0 sin 2 $\psi$ cos 3 $\varphi$	0 sin 3 $\psi$ cos 3 $\varphi$	0.25080956 cos $\psi$ cos 3 $\varphi$	0 cos 2 $\psi$ cos 3 $\varphi$	0.49203611 cos 3 $\psi$ cos 3 $\varphi$	
Ar 60°							
-461.42923611	0.70774964 sin $\psi$	-0.53799904 sin 2 $\psi$	0 sin 3 $\psi$	1.27475833 cos $\psi$	0.24865278 cos 2 $\psi$	4.09845833 cos 3 $\psi$	
1.1.0004952 sin $\varphi$	-2.59663333 sin $\psi$ sin $\varphi$	-1.39825000 sin 2 $\psi$ sin $\varphi$	0 sin 3 $\psi$ sin $\varphi$	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	-0.11041343 cos 3 $\psi$ sin $\varphi$	
-0.84856537 sin 2 $\varphi$	-1.52886667 sin $\psi$ sin 2 $\varphi$	-1.13351667 sin 2 $\psi$ sin 2 $\varphi$	0 sin 3 $\psi$ sin 2 $\varphi$	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	-0.05833643 cos 3 $\psi$ sin 2 $\varphi$	
0 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	0 sin 2 $\psi$ sin 3 $\varphi$	0 sin 3 $\psi$ sin 3 $\varphi$	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	0 cos 3 $\psi$ sin 3 $\varphi$	
2.35996389 cos $\varphi$	0.05304887 sin $\psi$ cos $\varphi$	0 sin 2 $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$	4.23794444 cos $\psi$ cos $\varphi$	1.63306111 cos 2 $\psi$ cos $\varphi$	0.56694722 cos 3 $\psi$ cos $\varphi$	
1.47113611 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 2 $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$	1.39283333 cos $\psi$ cos 2 $\varphi$	1.25217222 cos 2 $\psi$ cos 2 $\varphi$	0.84847500 cos 3 $\psi$ cos 2 $\varphi$	
4.48825278 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	0.06274354 sin 2 $\psi$ cos 3 $\varphi$	0 sin 3 $\psi$ cos 3 $\varphi$	0.27709722 cos $\psi$ cos 3 $\varphi$	0.23154722 cos 2 $\psi$ cos 3 $\varphi$	0.49203611 cos 3 $\psi$ cos 3 $\varphi$	

The objective of this paper is to illustrate a process that can be used to characterize these potential energy surfaces. Thus, rather than looking at the potential energy surface as a set of data points, we now want to look at the surfaces as a mathematical function.

## 2. Method

### 2.1. Molecular computations

Grid points at 15° increments were generated for both the *S* and *R* enantiomer of 1-chloro-3-fluoro-isobutane (Fig. 1) using Gaussian 98 program [1].

### 2.2. Multivariable Fourier expansion fitting of potential energy surface

A stepwise regression procedure was performed on grids of 15, 30, and 60° increments to fit a two independent variable ( $\varphi, \psi$ ) Fourier expansion to each surface. A functional analysis was then carried out on the three fitted surfaces of 28 function terms (including the constant) to determine the location of the minima from each expansion. Two independent variable ( $\varphi, \psi$ ) Fourier expansion of 32 function terms (including the constant) were fit using the same stepwise regression procedure to 30° increment grids for both enantiomers. A functional analysis was performed on both fitted 32 function term expansions.

Table 2  
Functional analysis fitted Fourier expansion of the potential energy surface

Conformer	$\varphi$	$\psi$	Energy (Hartree)	$\Delta E$ (kcal/mol)
<i>At 15° grid, 28 term expansion for S-1-chloro-3-fluoro-isobutane</i>				
$g^+g^+$	66.05	52.64	-711.4732988	-2.45418471
$g^+a$	60.73	173.04	-711.4708271	-0.90322534
$g^+g^-$	60.77	276.25	-711.4655433	2.412464705
$a g^+$	189.00	53.40	-711.4719539	-1.61026031
aa	189.07	181.09	-711.4693878	0
$ag^-$	185.60	298.65	-711.4715175	-1.33646013
$g^-g^+$	296.19	71.47	-711.4641447	3.29004748
$g^-a$	300.63	186.48	-711.4713441	-1.2276204
$g^-g^-$	295.18	299.85	-711.4735714	-2.62525962
<i>At 30° grid, 28 term expansion for S-1-chloro-3-fluoro-isobutane</i>				
$g^+g^+$	66.07	52.68	-711.473291	-2.45082188
$g^+a$	60.76	173.08	-711.4708144	-0.89667037
$g^+g^-$	60.86	276.21	-711.4655381	2.414241186
$a g^+$	189.06	53.41	-711.471967	-1.61993652
aa	189.09	181.08	-711.4693854	0
$ag^-$	185.60	298.62	-711.4715324	-1.34722632
$g^-g^+$	296.07	71.40	-711.4641311	3.297122655
$g^-a$	300.61	186.44	-711.4713289	-1.21955815
$g^-g^-$	295.19	299.78	-711.4735709	-2.62642302
<i>At 60° grid, 28 term expansion for S-1-chloro-3-fluoro-isobutane</i>				
$g^+g^+$	62.07	60.98	-711.47239	-1.88591981
$g^+a$	58.11	184.91	-711.4709272	0.387429694
$g^+g^-$	62.13	287.75	-711.4630391	3.981891688
$a g^+$	186.07	55.00	-711.471737	-1.47612942
aa	185.00	183.24	-711.4693846	0
$Ag^-$	182.60	302.19	-711.4715446	-1.35540905
$g^-g^+$	284.17	57.27	-711.463833	3.483676986
$g^-a$	297.48	181.97	-711.4715681	-1.37014361
$g^-g^-$	294.64	297.01	-711.4735979	-2.64387157

Table 3  
Tabulated components of the fitted 32 term Fourier expansion at 30°

<i>S enantiomer</i>			
-461.3098694	-0.65080338 sin $\psi$	0.56700006 sin 2 $\psi$	0.68923750 sin 3 $\psi$
-1.20991044 sin $\varphi$	-2.49149438 sin $\psi$ sin $\varphi$	-1.39762635 sin 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$
0.87788755 sin 2 $\varphi$	-1.45250963 sin $\psi$ sin 2 $\varphi$	-1.25563542 sin 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$
-0.64848333 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	-0.20055224 sin 2 $\psi$ sin 3 $\varphi$	-0.63070556 sin 3 $\psi$ sin 3 $\varphi$
2.57571291 cos $\varphi$	0 sin $\psi$ cos $\varphi$	0 sin 2 $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$
1.746222222 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 2 $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$
4.52506944 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	0 sin 2 $\psi$ cos 3 $\varphi$	0.15364722 sin 3 $\psi$ cos 3 $\varphi$
<i>R enantiomer</i>			
-461.3098694	0.65080338 sin $\psi$	-0.56700006 sin 2 $\psi$	-0.68923750 sin 3 $\psi$
1.20991044 sin $\varphi$	-2.49149438 sin $\psi$ sin $\varphi$	-1.39762635 sin 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$
-0.87788755 sin 2 $\varphi$	-1.45250963 sin $\psi$ sin 2 $\varphi$	-1.25563542 sin 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$
0.64848333 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	-0.20055224 sin 2 $\psi$ sin 3 $\varphi$	-0.63070556 sin 3 $\psi$ sin 3 $\varphi$
2.57571291 cos $\varphi$	0 sin $\psi$ cos $\varphi$	0 sin 2 $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$
1.746222222 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 2 $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$
4.52506944 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	0 sin 2 $\psi$ cos 3 $\varphi$	-0.15364722 sin 3 $\psi$ cos 3 $\varphi$
	1.32753090 cos $\psi$	0.36438819 cos 2 $\psi$	0.68923750 sin 3 $\psi$
	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$
	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$
	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	-0.63070556 sin 3 $\psi$ sin 3 $\varphi$
	4.41330725 cos $\psi$ cos $\varphi$	1.80465439 cos 2 $\psi$ cos $\varphi$	4.41330725 cos $\psi$ cos $\varphi$
	1.62978494 cos $\psi$ cos 2 $\varphi$	1.24906458 cos 2 $\psi$ cos 2 $\varphi$	1.62978494 cos $\psi$ cos 2 $\varphi$
	0.25080956 cos $\psi$ cos 3 $\varphi$	0.20336250 cos 2 $\psi$ cos 3 $\varphi$	0.25080956 cos $\psi$ cos 3 $\varphi$
	1.32753090 cos $\psi$	0.36438819 cos 2 $\psi$	0.68923750 sin 3 $\psi$
	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$
	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$
	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	-0.63070556 sin 3 $\psi$ sin 3 $\varphi$
	4.41330725 cos $\psi$ cos $\varphi$	1.80465439 cos 2 $\psi$ cos $\varphi$	4.41330725 cos $\psi$ cos $\varphi$
	1.62978494 cos $\psi$ cos 2 $\varphi$	1.24906458 cos 2 $\psi$ cos 2 $\varphi$	1.62978494 cos $\psi$ cos 2 $\varphi$
	0.25080956 cos $\psi$ cos 3 $\varphi$	0.20336250 cos 2 $\psi$ cos 3 $\varphi$	0.25080956 cos $\psi$ cos 3 $\varphi$
	1.32753090 cos $\psi$	0.36438819 cos 2 $\psi$	0.68923750 sin 3 $\psi$
	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$
	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$
	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	-0.63070556 sin 3 $\psi$ sin 3 $\varphi$
	4.41330725 cos $\psi$ cos $\varphi$	1.80465439 cos 2 $\psi$ cos $\varphi$	4.41330725 cos $\psi$ cos $\varphi$
	1.62978494 cos $\psi$ cos 2 $\varphi$	1.24906458 cos 2 $\psi$ cos 2 $\varphi$	1.62978494 cos $\psi$ cos 2 $\varphi$
	0.25080956 cos $\psi$ cos 3 $\varphi$	0.20336250 cos 2 $\psi$ cos 3 $\varphi$	0.25080956 cos $\psi$ cos 3 $\varphi$
	1.32753090 cos $\psi$	0.36438819 cos 2 $\psi$	0.68923750 sin 3 $\psi$
	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	-1.13824553 sin 3 $\psi$ sin $\varphi$
	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	-0.81786477 sin 3 $\psi$ sin 2 $\varphi$
	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	-0.63070556 sin 3 $\psi$ sin 3 $\varphi$
	4.41330725 cos $\psi$ cos $\varphi$	1.80465439 cos 2 $\psi$ cos $\varphi$	4.41330725 cos $\psi$ cos $\varphi$
	1.62978494 cos $\psi$ cos 2 $\varphi$	1.24906458 cos 2 $\psi$ cos 2 $\varphi$	1.62978494 cos $\psi$ cos 2 $\varphi$
	0.25080956 cos $\psi$ cos 3 $\varphi$	0.20336250 cos 2 $\psi$ cos 3 $\varphi$	0.25080956 cos $\psi$ cos 3 $\varphi$

### 2.3. Geometry optimization and subsequence analysis

The nine minima for each enantiomer, *S* and *R*, were optimized with the Gaussian 98 program [1] to determine a more precise location of the minima. For each enantiomer, the nine optimized minima were inserted into the corresponding 30° increment surface to obtain a new set of 153 grid points. A Fourier expansion was fitted to each set of 153 grid points followed by a functional analysis. These expansions have 33 function terms (including the constant). All of the stepwise regression procedures and functional analysis have been carried out using software [2,3].

### 3. Results and discussion

The computed grid points for both the *R* and *S* enantiomers are shown graphically in Fig. 2. The following Fourier expansion was the best equation for all the fitted surfaces:

$$E(\varphi, \psi) = c^0 + \sum_{m=0}^3 \sum_{n=0}^3 c_{m,n}^1 \cos m\varphi \cos n\psi \\ + c_{m,n}^2 \cos m\varphi \sin n\psi + c_{m,n}^3 \sin m\varphi \cos n\psi \\ + c_{m,n}^4 \sin m\varphi \sin n\psi$$

A systematic stepwise procedure was carried out to fit functions to the potential energy surface of 1-chloro-3-fluoro-isobutane for the *S* enantiomer at 15, 30, and 60° intervals to see the similarities and differences amongst the equations. The idea is to have the most accurate fit with the least amount of terms. In the regression analysis for the 60° refinement, only the most important 28 terms (including the constant) were kept. In order to compare the 15, 30 and 60° grid refinements 28 term expression had to be used for 15 and 30° refinement to match the limitation of the 60° refinement. The tabulated components of the fitted Fourier expansion for 15, 30, and 60° are found in Table 1. A functional analysis was carried out on the three fitted equations at 15, 30, and 60° increments found in Table 2. The first derivative of the equations was solved for the location of the critical points where nine minima were located for each surface. The Fourier expansion that seemed to be the best choice is found in Table 1.

Table 4  
Functional analysis of 32 term fitted Fourier expansion

Conformer	$\varphi$	$\psi$	Energy (Hartree)	$\Delta E$ (kcal/mol)
<i>S</i> enantiomer				
$g^+g^+$	65.06	52.83	-711.4730849	-2.24712084
$g^+a$	60.34	173.86	-711.4709577	-0.912308551
$g^+g^-$	61.00	276.78	-711.465195	2.703853994
$a g^+$	187.79	53.35	-711.4717184	-1.389646623
aa	187.79	181.31	-711.4695038	0
$ag^-$	185.94	300.08	-711.4714284	-1.207654918
$g^-g^+$	291.83	68.10	-711.4641364	3.368130432
$g^-a$	299.69	186.92	-711.4715779	-1.301497783
$g^-g^-$	295.05	300.84	-711.4734743	-2.49150712
<i>R</i> enantiomer				
$g^+g^+$	64.95	59.16	-711.4734743	-2.49150712
$g^+a$	60.31	173.08	-711.4715779	-1.301497783
$g^+g^-$	68.17	291.90	-711.4641364	3.368130432
$ag^+$	174.06	59.92	-711.4714284	-1.207654918
aa	172.21	178.69	-711.4695038	0
$ag^-$	172.21	306.65	-711.4717184	-1.389646623
$g^-g^+$	299.00	83.22	-711.465195	2.703853994
$g^-a$	299.66	186.71	-711.4709577	-0.912308551
$g^-g^-$	294.94	307.17	-711.4730849	-2.24712084

Table 5  
Nine optimized minima

Conformer	$\varphi$	$\psi$	Energy (Hartree)	$\Delta E$ (kcal/mol)
<i>S</i> enantiomer				
$g^+g^+$	66.61	53.69	-711.4729679	-2.160287261
$g^+a$	61.23	175.43	-711.4709131	-0.870928032
$g^+g^-$	63.37	278.24	-711.4654592	2.551454405
$a g^+$	188.75	53.99	-711.4721491	-1.646524117
aa	187.81	181.41	-711.4695252	0
$ag^-$	185.88	301.65	-711.4716381	-1.325840151
$g^-g^+$	283.42	61.97	-711.4645667	3.111510845
$g^-a$	296.27	184.39	-711.4717223	-1.378718536
$g^-g^-$	290.98	301.11	-711.4739387	-2.769498442
<i>R</i> enantiomer				
$g^+g^+$	69.02	58.89	-711.4739387	-2.769498442
$g^+a$	63.73	175.61	-711.4717223	-1.378718536
$g^+g^-$	76.58	298.03	-711.4645667	3.111510845
$a g^+$	174.12	58.35	-711.4716381	-1.325840151
aa	172.19	178.59	-711.4695252	0
$ag^-$	171.25	306.01	-711.4721491	-1.646524117
$g^-g^+$	296.63	81.76	-711.4654592	2.551454405
$g^-a$	298.77	184.57	-711.4709131	-0.870928032
$g^-g^-$	293.39	306.31	-711.4729679	-2.160287261

Table 6  
Tabulated components of the fitted 33 term Fourier expansion at 30° with the optimized minima included

<i>S</i> enantiomer					
-461.3195342	0.65354330 sin $\psi$	-0.69250128 sin 3 $\psi$	1.32093506 cos $\psi$	0.37267006 cos 2 $\psi$	4.18658806 cos 3 $\psi$
1.21766019 sin $\varphi$	-2.48509560 sin $\psi$ sin $\varphi$	-1.14481608 sin 3 $\psi$ sin $\varphi$	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	-0.11493268 cos 3 $\psi$ sin $\varphi$
-0.87603883 sin 2 $\varphi$	-1.45079124 sin $\psi$ sin 2 $\varphi$	-0.82457165 sin 3 $\psi$ sin 2 $\varphi$	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	0 cos 3 $\psi$ sin 2 $\varphi$
0.64586732 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	-0.62760531 sin 3 $\psi$ sin 3 $\varphi$	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	0.17101827 cos 3 $\psi$ sin 3 $\varphi$
2.57723405 cos $\varphi$	0 sin $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$	4.41970684 cos $\psi$ cos $\varphi$	1.80377986 cos 2 $\psi$ cos $\varphi$	0.52750387 cos 3 $\psi$ cos $\varphi$
1.74986802 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$	1.62744740 cos $\psi$ cos 2 $\varphi$	1.24472336 cos 2 $\psi$ cos 2 $\varphi$	0.69766075 cos 3 $\psi$ cos 2 $\varphi$
4.54189872 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	-0.14711437 sin 3 $\psi$ cos 3 $\varphi$	0.26154371 cos $\psi$ cos 3 $\varphi$	0.18927849 cos 2 $\psi$ cos 3 $\varphi$	0.46272898 cos 3 $\psi$ cos 3 $\varphi$
<i>R</i> enantiomer					
-461.3195342	-0.65354330 sin $\psi$	0.69250128 sin 3 $\psi$	1.32093506 cos $\psi$	0.37267006 cos 2 $\psi$	4.18658806 cos 3 $\psi$
-1.21766019 sin $\varphi$	-2.48509560 sin $\psi$ sin $\varphi$	-1.14481608 sin 3 $\psi$ sin $\varphi$	0 cos $\psi$ sin $\varphi$	0 cos 2 $\psi$ sin $\varphi$	0.11493268 cos 3 $\psi$ sin $\varphi$
0.87603883 sin 2 $\varphi$	-1.45079124 sin $\psi$ sin 2 $\varphi$	-0.82457165 sin 3 $\psi$ sin 2 $\varphi$	0 cos $\psi$ sin 2 $\varphi$	0 cos 2 $\psi$ sin 2 $\varphi$	0 cos 3 $\psi$ sin 2 $\varphi$
-0.64586732 sin 3 $\varphi$	0 sin $\psi$ sin 3 $\varphi$	-0.62760531 sin 3 $\psi$ sin 3 $\varphi$	0 cos $\psi$ sin 3 $\varphi$	0 cos 2 $\psi$ sin 3 $\varphi$	-0.17101827 cos 3 $\psi$ sin 3 $\varphi$
2.57723405 cos $\varphi$	0 sin $\psi$ cos $\varphi$	0 sin 3 $\psi$ cos $\varphi$	4.41970684 cos $\psi$ cos $\varphi$	1.80377986 cos 2 $\psi$ cos $\varphi$	0.52750387 cos 3 $\psi$ cos $\varphi$
1.74986802 cos 2 $\varphi$	0 sin $\psi$ cos 2 $\varphi$	0 sin 3 $\psi$ cos 2 $\varphi$	1.62744740 cos $\psi$ cos 2 $\varphi$	1.24472336 cos 2 $\psi$ cos 2 $\varphi$	0.69766075 cos 3 $\psi$ cos 2 $\varphi$
4.54189872 cos 3 $\varphi$	0 sin $\psi$ cos 3 $\varphi$	0.14711437 sin 3 $\psi$ cos 3 $\varphi$	0.26154371 cos $\psi$ cos 3 $\varphi$	0.18927849 cos 2 $\psi$ cos 3 $\varphi$	0.46272898 cos 3 $\psi$ cos 3 $\varphi$

The 30° grid was chosen to be the best comprise.

When the *S* and *R* isomers were studied in the 30° refinement it was possible to use a more extensive expansion as was used before. In the present case the regression analysis at the 30° refinement allowed a 32-term expansion (including the constant). The tabulated components for the 32 term Fourier expansion are shown in Table 3. There is a sign change between the *R* and *S* for the odd function terms, while there is no sign change for the even function terms. The Functional analysis for the 32 expansions can be found in Table 4.

Gaussian gradient optimizations were performed at the minimum energy points of this function and the results are compared to the minima values found from the fitted function (Table 5). The nine optimized structures were added to the 30° refinement the regression analysis allowed the inclusion of 33 terms (including the constant). The tabulated components of the fitted 33 terms Fourier expansion for both the *S* and *R* enantiomer is found in Table 6. A functional analysis was performed on the 33 term function for both enantiomers found on Table 7.

Table 7  
Functional Analysis of 33 term fitted Fourier expansion

Conformer	$\varphi$	$\psi$	Energy (Hartree)	$\Delta E$ (kcal/mol)
<i>S</i> enantiomer				
$g^+g^+$	64.98	52.73	-711.4730625	-2.210913513
$g^+a$	60.23	173.67	-711.4708962	-0.851530442
$g^+g^-$	60.90	276.36	-711.4652511	2.690820611
$a g^+$	187.86	53.41	-711.4718174	-1.429563162
aa	187.91	181.26	-711.4695392	0
$ag^-$	185.99	300.01	-711.4715289	-1.248559785
$g^-g^+$	290.88	66.97	-711.4643434	3.26039073
$g^-a$	299.52	186.66	-711.4717155	-1.365678878
$g^-g^-$	294.99	300.79	-711.4736778	-2.596983393
<i>R</i> enantiomer				
$g^+g^+$	65.01	59.21	-711.4736778	-2.596983393
$g^+a$	60.48	173.34	-711.4717155	-1.365678878
$g^+g^-$	69.12	293.03	-711.4643434	3.26039073
$a g^+$	174.01	59.99	-711.4715289	-1.248559785
aa	172.09	178.74	-711.4695392	0
$ag^-$	172.14	306.59	-711.4718174	-1.429563162
$g^-g^+$	299.11	83.64	-711.4652511	2.690820611
$g^-a$	299.77	186.33	-711.4708962	-0.851530442
$g^-g^-$	295.02	307.27	-711.4730625	-2.210913513

The accuracy of the  $\varphi$  and  $\psi$  values obtained from the functional analysis of the Fourier expansion can be assessed when compared to the  $\varphi$  and  $\psi$  values obtained by direct optimization. The comparison of

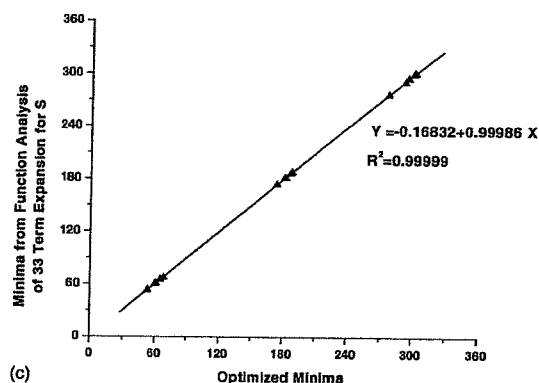
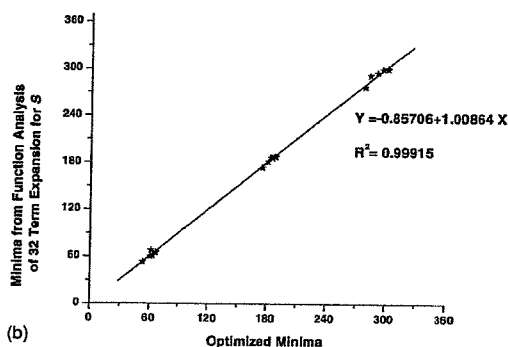
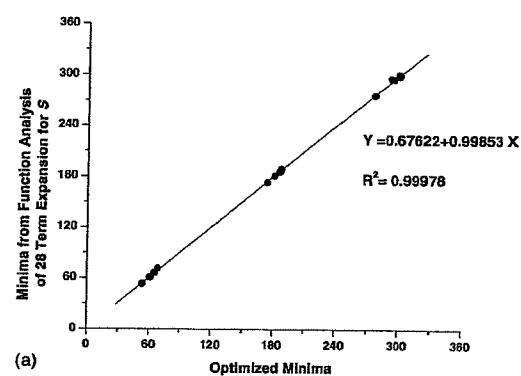


Fig. 3. (a) Minima from Functional Analysis of 27 term expansion for *S* vs. optimized minima. (b) Minima from Functional Analysis of 31 term expansion for *S* vs. optimized minima. (c) Minima from Functional Analysis of 32 term expansion for *S* vs. optimized minima.



the data presented at Table 2 against Table 5 as well on Table 4 against Table 5 and Table 7 against Table 5 are shown graphically in Fig. 3.

#### 4. Conclusions

It appears that 30° resolution of grid is satisfactory to determine the location of the minima ( $\varphi, \psi$ ) with respect to the optimized structures with reasonably degree accuracy (i.e.  $R^2 = 0.99637582$ ). It seems that the inclusion of the optimized minima in the generated grid with not improve dramatically the results obtained by the functional analysis.

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