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PREFACE

Since the early 70's, there has been intensive development in the theory of functions of an infinite number of complex variables. This has led to the establishment of completely new principles (e.g. concerning the behaviour of fixed points) and has thrown new light on some classical finite dimensional results such as the maximum principle, the Schwarz lemma and so on. Perhaps the most spectacular advances occurred in connection with the old problem of the determination of the holomorphic automorphisms of complex manifolds.

This book is based on the introductory lectures on this latter field delivered at the University of Santiago de Compostela in October 1981 by the authors. Originally, it was planned as a comprehensive postgraduate course relying on a deep knowledge of holomorphy in topological vector spaces and infinite dimensional Lie groups. However, seeing that some of the undergraduate students were mainly interested in the study of bounded domains in Banach spaces, the authors restricted their attention to these aspects. This proved to be a fortunate idea. We realized that by combining the methods of the theories developed independently by W. Kaup and J.P. Vigué with minor modifications, even the main theorems could be derived. This was achieved in a self-contained way from the most fundamental principles of Banach spaces (such as the open mapping theorem), elementary function theory and the pure knowledge of the Taylor series representation of holomorphic maps in this setting. It may often happen in teaching mathematics that avoiding the introduction of strong tools leads to abandoning natural heuristics. Probably, this is not the case now. It is enough to recall how deeply the early development of the theory of finite dimensional Lie groups and Lie algebras was inspired in Cartan's investigation of the structure of symmetric domains. Moreover, we think that this approach to the automorphism groups of Banach space domains may also serve as motivating and illustrative material in introducing students to the theory of Lie groups and complex manifolds.

The text is divided into eleven chapters. In chapter 0 we establish the terminology, and some typical examples of later importance (e.g. the Möbius group) are studied. In chapter 1 we show the main topological consequences of the Cauchy estimates of Taylor coefficients for uniformly bounded families of holomorphic mappings. These considerations are continued in chapter 2 and applied specifically to the case of the automorphism group, concluding with the topological version of Cartan's uniqueness theorem. The global topological investigations finish in chpater 3, where the Carathéodory distance is introduced to obtain the completness properties of the group AutD. In chapter 4 a completely elementary introduction to Lie theory begins by showing where one-parameter subgroups come from. Chapter 5 is devoted to a description of the Banach Lie algebra structure of complete holomorphic vector fields in order to lay the foundation of chpater 6, in which the Banach Lie groups structure of AutD is studied. In chpaters 7 and 8 we discuss the basic theory of circular domains and determine explicitly the holomorphic automorphism group of the unit ball of several classical Banach spaces. In chapter 9 we introduce the reader to another fruitfully developing branch of these researches by proving Vigué's theorem on the Harish-Chandra realization of bounded symmetric domains. Finally, in chapter 10 and elementary introduction of the Jordan approach to bounded symmetric domains is presented and the convexity of the Harish-Chandra realization is proved.

We would like to express our sincere acknowledgement to Prof. L. Nachbin who suggested the idea of writing these notes and who, together with Prof. E. Vesentini, introduced the authors to infinite dimensional holomorphy and this fascinating branch of mathematics.

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The authors, August 1984.

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