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Bicircular projections and characterization of Hilbert spaces. (English summary)
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The topic of this paper is motivated by the investigation of the same authors on continuous Reinhardt domains. These are open subsets of $\mathcal{C}_0(\Omega)$ satisfying certain additional conditions. In the study of Reinhardt domains one is led to showing that the kernel of certain projections with one-dimensional range and an additional norm property, which is called bicircularity, are Hilbert spaces.

However, the notion of bicircularity makes sense in a purely Banach space theoretic setting. Let X be a complex Banach space in some norm $\|\cdot\|$, and let $P\colon X\to X$ be a bounded linear projection. We say that P is bicircular if the mappings of the form $e^{i\alpha}P+e^{i\beta}(1-P)$ are isometric for all pairs of real numbers α,β. A natural problem is to describe the bicircular projections of a given Banach space. Clearly, the answer depends on the norm of the space and can change if an equivalent norm is taken instead.

In a recent paper, the authors have studied in detail this type of projections in the classical spaces $B(H)$, $S(H)$ and $A(H)$ consisting respectively of all bounded linear, all symmetric bounded linear and all antisymmetric bounded linear operators $X\colon H\to H$, where H is a complex Hilbert space. Thus what was initially a technical point in the study of Reinhardt domains has been expanded into an independent study in its proper algebraic setting, which is that of JB*-triples. Here the authors prove that the bicircular projections of a JB*-triple X are exactly the skew derivations X. The main result of the paper is a characterization of Hilbert spaces among JB*-triples. Namely the authors establish that given a JB*-triple X and a bicircular projection of rank one $P\colon X\to X$, there are two closed ideals J and H of X such that $X=J\oplus H$, where H is isomorphic to a Hilbert space, $P(X)\subset H$ and $J\subset\ker(P)$. As a corollary, Hilbert spaces are precisely the only prime JB*triples that admit rank-one bicircular projections.

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