MR1778402 (2001g:46103) 46G20 (46M20 58B12)
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Holomorphic automorphisms of continuous products of balls. (English summary)

The authors consider “continuous products” \(D\) of bounded domains \(D_\omega\) in a Banach space \(E\) indexed by \(\omega \in \Omega = \) a compact Hausdorff space. One result establishes conditions on \(D_\omega\) so that every complete holomorphic vector field on the product is “fibre preserving”.

For the case where each \(D_\omega\) is the unit ball for some equivalent norm on \(E\), the orbit of the origin under biholomorphic automorphisms of \(D\) is characterised via the corresponding orbits in each \(D_\omega\) and a continuity condition for the coordinate-wise triple product with the middle variable fixed. This result can be applied to give a simple form for the case when \(D_\omega\) is independent of \(\omega\).

Another case investigated is where there is a family of bounded surjective linear isomorphisms \(\varphi_\omega\) of \(E\), norm bounded above and with inverse bounded above in norm, so that \(D_\omega\) is the image of the unit ball of \(E\) under \(\varphi_\omega\). Assume \((x, \omega) \in E \times \Omega \mapsto \varphi_\omega(x)\) is upper semicontinuous on the product space, and that the unit ball of \(E\) is a symmetric domain. Then the orbit of the origin in \(D\) is shown to contain a part of \(D\) with a vanishing condition at points of discontinuity of \(\omega \in \Omega \mapsto \varphi_\omega\) (for the strong operator topology on linear self-maps of \(E\)). In an example of a product of two-dimensional Hilbert balls, the containment is strict. In the case where each \(\varphi_\omega\) is a positive multiple of the identity, the vanishing condition gives the precise orbit (recovering a result of J.-P. Vigué [Ark. Mat. 36 (1998), no. 1, 177–190; MR1611169 (99b:58018)])

Reviewed by Richard M. Timoney

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.