MR0442202 (56 #588) 28A75 (52A20)
Stachó, L. L.
On the volume function of parallel sets.
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Call a continuous real-valued function $f(t)$, $t>0$, a Kneser function of order n if, for $b\geq a>0$ and $\lambda\geq 1$, $f(\lambda b)-f(\lambda a)\leq\lambda^n[f(b)-f(a)]$. An interesting integral characterization of such functions is established: $f(t)$ is a Kneser function of order n if and only if there exists a monotone decreasing function α such that $f(t)=\int_\alpha^t\tau^{n-1}\alpha(\tau)\,d\tau+f(a)$. This yields an integral representation of the volume function of parallel sets in Euclidean space E^n. For if $K\subset E^n$ is a bounded open centrally symmetric convex set and $A\subset E^n$ is bounded, then the author shows that the volume function $|A+tK|$, $t>0$, is a Kneser function of order n. This was previously proved by M. Kneser [Math. Nachr. **5** (1951), 241–251; MR0042729 (13,154c)] for $V(t)=|A+tU|$, where U is the open unit ball in E^n. Generalizing a result of C. Pucci [Boll. Un. Mat. Ital. (3) **12** (1957), 420–421; MR0089887 (19,734f)], the author also shows that the $(n-1)$-dimensional measure of the boundary of $A+tU$, $t>0$, is the arithmetic mean of the left- and right-hand derivatives of $V(t)$.

Reviewed by *C. M. Petty*