On a Schwarz Lemma for Bounded Symmetric Domains

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ABSTRACT.

1. Introduction

It is well known that the classical Schwarz Lemma allows the following higher dimensional extension: Let E, F be complex Banach spaces with open unit balls $B \subset E$, $D \subset F$ and let $f: B \to D$ be a holomorphic mapping with $f(0) = 0$. Then $\|f(z)\| \leq \|z\|$ for all $z \in B$ and $\|L\| \leq 1$ hold, where the linear operator $L: E \to F$ is the complex derivative at 0 of f. But in contrast to the classical case $E = F = \mathbb{C}$, the condition $\|f(z)\| = \|z\|$ for some $z \neq 0$ and also the condition $\|L\| = 1$ does in general not imply that f is linear (more precisely the restriction to B of a linear map – necessarily the derivative $L = df(0)$). To have a short notation we call the ordered pair of complex Banach spaces (E, F) *rigid* if every holomorphic mapping $f: B \to D$ with $f(0) = 0$ is linear provided that the derivative $df(0): E \to F$ is a (not necessarily surjective) isometry. In case this conclusion already follows without the assumption $f(0) = 0$ we call the pair *strictly rigid*. For instance, (E, F) is rigid if every unit vector in F is a complex extremal boundary point of D and this condition is also necessary if $E = \mathbb{C}$, compare [1]. Also, (E, E) is strictly rigid for every complex Banach space E of finite dimension as a consequence of Cartan's uniqueness theorem, compare [5] and [1]. The rigidity condition for (E, F) is not symmetric in E, F. In particular, (E, F) trivially is rigid if there is no linear isometry $E \to F$.

Suppose that $\mathcal{K}$ is a class of complex Banach spaces and that $\varphi: \mathcal{K} \to \mathbb{N} \cup \{\infty\}$ is a function. We will consider the following property for φ.
Property A: For all $E, F \in \mathcal{K}$ with $\varphi(F) \leq \varphi(E) < \infty$ the pair (E, F) is rigid.
Since for spaces with φ-value ∞ nothing is claimed in this property we always may assume without loss of generality that $\mathcal{K}$ is the class $\mathcal{B}$ of all complex Banach spaces (simply by extending φ using the value ∞). For instance, on $\mathcal{B}$ the function $\varphi = \dim$ satisfies Property A. But also the following function ψ satisfies Property A: For every complex Hilbert space E put $\psi(E) = 1$. In case E is not a Hilbert space but every unit vector is an extreme point of its unit ball put $\psi(E) = 2$. In the remaining cases put $\psi(E) = \infty$. Clearly, this would be more interesting if some of the values ∞ could be changed to a finite one while keeping Property A.

In the present paper we consider certain rank functions with Property A on the class of complex Banach spaces associated with bonded symmetric domains. It is known that every bounded symmetric domain in a complex Banach space can be realized as the open unit ball of another complex Banach space E uniquely determined up to linear isometry [7]. These Banach spaces are called JB*-triples since they may be algebraically characterized by a certain ternary structure, the Jordan triple product.

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2. The rank function

Fix the field $\mathbb{K}$ in the following which is either $\mathbb{R}$ or $\mathbb{C}$. Denote by $\mathcal{B}$ the category of all $\mathbb{K}$-Banach spaces with the bounded $\mathbb{K}$-linear mappings as morphisms. Throughout, E and F are Banach spaces with open unit balls $B \subset E$ and $D \subset F$. The notation $E \subset F$ means that E carries the induced norm from F, i.e. $B = D \cap F$. Also we write $E \preccurlyeq F$ to indicate that there exists a (not necessarily surjective) linear isometry $E \to F$. By $\mathcal{L}(E,F)$ we denote the Banach space of all bounded linear operators $E \to F$. Furthermore $\mathcal{L}(E) := \mathcal{L}(E,E)$ is the Banach algebra of all continuous endomorphisms and $E^* := \mathcal{L}(E,\mathbb{C})$ is the dual of E. The group of all invertible operators in $\mathcal{L}(E)$ is denoted by $\mathsf{GL}(E)$. The vector space dimension of E over $\mathbb{K}$ is denoted by $\dim(E)$ and will be considered as an element of $\overline{\mathbb{N}} := \mathbb{N} \cup \{\infty\}$.

The boundary of B (the unit sphere in E) is denoted by ∂B. The subset of all extreme boundary points of B is denoted by $\partial_e B$, that is the set of all $a \in \partial B$ with the property: $\|a \pm v\| = 1$ implies $v = 0$ for all $v \in E$. In the complex case (i.e. $\mathbb{K} = \mathbb{C}$) the point $a \in \partial B$ is called complex extreme if $\|a + tv\| = 1$ for all $t \in \Delta$ always implies $v = 0$, where $\Delta \subset \mathbb{C}$ is the open unit disc. With $\partial_{ce} B \subset \partial B$ we denote the subset of all complex extreme boundary points. Also we denote for every complex Banach space E by $E^{\mathbb{R}}$ the underlying real Banach space. Clearly, E and $E^{\mathbb{R}}$ have to be distinguished, for instance $\dim(E^{\mathbb{R}}) = 2\dim(E)$ holds in our notation.

We are interested in functions $\varphi: \mathcal{B} \to \overline{\mathbb{N}}$ satisfying
Property B: $\varphi(E) \le \varphi(F)$ for all $E, F \in \mathcal{B}$ with $E \preccurlyeq F$.

It is clear that $\varphi = \dim$ satisfies this property. Further examples can be obtained in the following way: Let φ satisfy Property B. For every Banach space E and every $a \in E$ let Θ_a be the closed linear span of

$$\{v \in E : \|a + tv\| = \|a\| \text{ for all } t \in \mathbb{K} \text{ with } |t| \le 1\}$$

in E (this notion coincides except for $a = 0$ with the one in [1]). Then $\varphi'(E) := \sup_{a\in E} \varphi(\Theta_a)$ defines a function $\varphi': \mathcal{K} \to \overline{\mathbb{N}}$, and it is clear that every linear isometry $L: E \to F$ maps Θ_a into $\Theta_{L(a)}$. Therefore, with φ also φ' satisfies Property A. We call φ' the *derived function of* φ. Then $\varphi' \le \varphi$ is easily seen and by iteration we also get φ'' and so on. As an example, $\dim'(E) = 0$ holds if and only if $\partial B = \partial_e B$ in case $\mathbb{K} = \mathbb{R}$ and $\partial B = \partial_{ce} B$ in case $\mathbb{K} = \mathbb{C}$. Also, if $E = \mathcal{L}(H,K)$ for Hilbert spaces H, K with $\dim(H) = 2$ and $\dim(K) = n \ge 1$ we have $\dim(E) = 2n$, $\dim'(E) = n - 1$ and $\dim''(E) = 0$ (even if n is infinite).

For every E put furthermore $r_\varphi(E) := \inf\{n \in \mathbb{N} : \varphi^{(n)}(E) = 0\}$, where $\varphi^{(n)}$ is the n-th derivative of φ and $\inf \emptyset = \infty$. In case of $\varphi = \dim$ we also write $r(E) = r_{\dim}(E)$ and call it the *rank* of the Banach space E. The following statement is easily verified.

2.1 Lemma. *With φ also all derivatives $\varphi^{(n)}$ and also r_φ satisfies* Property B.

In particular, the rank r satisfies Property B. All elements of $\overline{\mathbb{N}}$ occur as a rank: Consider for example the Banach space $E = \mathcal{C}_0(S,\mathbb{K})$ of all $\mathbb{K}$-valued continuous functions vanishing at infinity on the locally compact topological space S. Then it is not difficult to see that $r(E) = \dim(E) = |S|$ where $|S| \in \overline{\mathbb{N}}$ is the number of elements in S. Actually, we can show a little bit more. Denote by $E \oplus_p F$ the ℓ^p-sum of E and F, that is $E \oplus F$ with norm satisfying $\|(z,w)\| = \max(\|z\|, \|w\|)$ if $p = \infty$ and $\|(z,w)\|^p = \|z\|^p + \|w\|^p$ if $1 \le p < \infty$. Instead of $E \oplus_\infty F$ we also write $E \times F$ since then the open unit ball is $B \times D$.

2.2 Proposition. *For all Banach spaces E, F the following statements hold.*
(i) $r(E) \le \dim(E)$ *and* $r(E) = 0$ *if and only if* $E = \{0\}$.
(ii) $\sup_{a\in E} r(\Theta_a) = r(E) - 1$ *if* $E \ne \{0\}$.
(iii) $r(E \times F) = r(E) + r(F)$.

Proof. (i) is obvious.
(ii) We may assume that $k := \sup_{a\in E} r(\Theta_a) < \infty$ since $k \le r(E)$. For $\varphi := \dim$ this means

$$\varphi^{(k+1)}(E) = \sup_{a\in E} \varphi^{(k)}(\Theta_a) = 0,$$

i.e. $r(E) \leq k+1$. But $r(E) \leq k$ would contradict the definition of k.
(iii) We assume that $0 < r(E) \leq r(F)$ holds and use induction on $n = r(E) + r(F)$. The case $n = 0$ is trivial and for $n = \infty$ the statement follows from $r(E \times F) \geq r(F) = \infty$. Therefore we only have to consider the case $0 < n < \infty$. For all $(a,b) \in E \times F$

$$\Theta_{(a,b)} = \begin{cases} \Theta_a \times F & \|a\| > \|b\| \\ \Theta_a \times \Theta_b & \|a\| = \|b\| \\ E \times \Theta_b & \|a\| < \|b\| \end{cases}$$

is easily seen. Then by induction hypothesis we have

$$\sup_{(a,b)\in E\times F} r\big(\Theta_{(a,b)}\big) = n-1 \quad \text{and hence} \quad r(E\times F) = n \quad \text{by (ii).} \qquad \square$$

Property (ii) implies that the n-th derivative $r^{(n)}$ of the rank function does not give further information since

$$r^{(n)} = \max(r-n, 0) \quad \text{for all} \quad n \in \mathbb{N}\,.$$

For the rest of the paper let $\mathbb{K}$ be the complex field. For every complex Banach space E then let $\rho(E) := r(E^{\mathbb{R}})$ be the *real rank* of E. Then it is clear that also the function ρ on $\mathcal{B}$ satisfies Property B. Also, by induction it can be shown that always $r(E) \leq \rho(E)$ holds. The question arises: To what extent do the rank functions r and ρ satisfy Property A? In the next section we prove this for the class of JB*-triples.

For certain complex Banach spaces E of finite dimension Vigué [12] has defined a rank $r(B)$ of the open unit ball $B \subset E$. Since in case that B is a bounded symmetric domain this rank in general is not the usual one, we prefer to write $r_{\mathcal{V}}(E)$ instead of $r(B)$ here. Let $\mathcal{V}$ be the class of all complex Banach spaces of finite dimension such that the set

$$\{x \in E : \dim \Theta_x = \sup_{a\in E} \dim \Theta_a = \dim'(E)\}$$

is dense in E. Then $r_{\mathcal{V}}(E) = 1 + \dim'(E)$ in our language and the result in [12], Théorème 5.2, can be expressed in the following way: *The function $r_{\mathcal{V}}$ on $\mathcal{V}$ satisfies* Property A.

3. JB*-triples

For complex Banach spaces E, F with open unit balls B, D a mapping $f\colon B \to D$ is called *holomorphic* if for every $a \in B$ the Fréchet derivative $df(0) \in \mathcal{L}(E,F)$ exists. The holomorphic mapping f is called biholomorphic if the inverse mapping $D \to B$ exists and is holomorphic. Cartan's uniqueness theorem states that for every $a \in B$ every biholomorphic map $f\colon B \to D$ is uniquely determined within the space of all holomorphic mappings $B \to D$ by $f(a)$ and $df(a)$ (compare f.i. [4] p. 75). With $\mathrm{Aut}(B)$ we denote the group of all biholomorphic mappings $g\colon B \to B$, also called biholomorphic automorphisms of B.

The complex Banach space E is called a *JB*-triple* if the group $\mathrm{Aut}(B)$ acts transitively on the open unit ball B. To every $a \in B$ then there is a unique automorphism $s_a \in \mathrm{Aut}(B)$ with $s_a = s_a^{-1}$, $s_a(a) = a$ and $ds_a(a) = -\mathrm{id}$, i.e. D is a bounded symmetric domain. Denote by $\mathcal{JB}$ the category of all JB*-triples. By definition a linear map $L\colon E \to F$ is a morphism in $\mathcal{JB}$ if $L \circ s_a = s_c \circ L$ holds for all $a \in B$ and $c := L(a) \in D$. It is clear that with E, F also the ℓ^∞-sum $E \times F$ is in $\mathcal{JB}$ and that the canonical projections are triple morphisms. JB*-triples can also be introduced without any reference to holomorphy by the existence of a Jordan triple product $(a,b,c) \mapsto \{abc\}$ from E^3 to E that is symmetric complex bilinear in the outer variables a, c and conjugate linear in the middle variable b together with some other properties, compare [7]. For instance, for every pair H, K of complex Hilbert spaces every closed linear subspace $E \subset \mathcal{L}(H,K)$ stable under the triple product $\{abc\} = (ab^*c + cb^*a)/2$ is a JB*-triple. Therefore, every C*-algebra and also every complex Hilbert space is in $\mathcal{JB}$, where in the latter case $\{aba\} = (a|b)a$ holds. The morphisms in $\mathcal{JB}$ can also been characterized algebraically by the triple product:

The linear map $L: E \to F$ is a triple morphisms if and only if $L\{abc\} = \{L(a)L(b)L(c)\}$ holds for all $a, b, c \in E$. Triple morphisms always have closed range and are automatically continuous (the induced map $E/\ker(L) \to F$ is an isometry). On the other hand, every surjective linear isometry in $\mathcal{JB}$ is a triple isomorphism.

Let E, F always be JB*-triples in the following. For every $a, b \in E$ denote the linear operator $z \mapsto \{abz\}$ by $a \square b$. Then $\|a \square b\| \leq \|a\| \cdot \|b\|$ holds and $\square$ may be considered as an operator-valued inner product on E. We write $a \perp b$ and call a, b orthogonal if $\|a \square b\| = 0$ or – equivalently – if $\|b \square a\| = 0$ holds. For every $a \in E$ and $n \in \mathbb{N}$ the odd powers are defined by $a^{2n+1} := (a \square a)^n a$. These always satisfy $\|a^{2n+1}\| = \|a\|^{2n+1}$. It is clear that the triple product on E is uniquely determined by the cube mapping $a \mapsto a^3 = \{aaa\}$. The fixed points of the cube mapping are called *tripotents*. The set $M \subset E$ of all tripotents is a real analytic submanifold of E and every non-zero tripotent $e \in E$ has norm 1. Suppose $e_1, \ldots, e_r$ are pairwise orthogonal tripotents in E. Then for every $i, j \in \{0, 1, \ldots, r\}$ the Peirce space

$$E_{ij} := E_{ij}(e_1, \ldots, e_r) := \{z \in E : 2\{e_k e_k z\} = (\delta_{ik} + \delta_{jk})z \text{ for all } k\}$$

is a subtriple with $E_{ij} = E_{ji}$ and

$$E = \bigoplus_{0 \leq i \leq j \leq r} E_{ij}$$

is called the corresponding Peirce decomposition, compare [10]. The Peirce spaces multiply according to the rules

$$\{E_{ij} E_{jk} E_{kl}\} \subset E_{il} \qquad \text{and} \qquad E_{ij} \square E_{pq} = 0 \quad \text{if} \quad i, j \notin \{p, q\}.$$

In particular, we have the Peirce decomposition $E = E_{11}(e) \oplus E_{10}(e) \oplus E_{00}(e)$ for every single tripotent $e \in E$. The tripotent e is called *minimal in* E if $\dim(E_{11}(e)) = 1$ holds.

For every $a \in E$ denote by $E_a \subset E$ the smallest closed subtriple of E that contains a and put $d(a) := \dim(E_a) \in \overline{\mathbb{N}}$. It is known that E_a is isometrically isomorphic to $\mathcal{C}_0(S) := \mathcal{C}_0(S, \mathbb{C})$ for some locally compact topological space S. In particular, also $d(a) = r(E_a)$ holds where $r(E_a)$ is the Banach space rank as defined in the previous section. By definition, the triple rank of E is the supremum in $\overline{\mathbb{N}}$ of all $d(a)$ with $a \in E$.

3.1 Proposition. *For every JB*-triple E the triple rank and the Banach space rank $r(E)$ coincide.*

Proof. Denote for a while the triple rank of E by $\widetilde{r}(E)$. We have to show $\widetilde{r}(E) = r(E)$. For every $a \in E$ we have $d(a) = r(E_a) \leq r(E)$ and hence $\widetilde{r}(E) \leq r(E)$. Therefore we may assume that $n := \widetilde{r}(E) < \infty$ holds. In case $n = 0$ we have $E = 0$, i.e. in addition we may assume $n > 0$. For every $a \in E$ with $a \neq 0$ there exists a unique representation

$$a = \lambda_1 e_1 + \cdots + \lambda_s e_s \quad \text{with} \quad \lambda_1 > \lambda_2 > \cdots > \lambda_s > 0,$$

where $e_1, e_2, \cdots, e_s$ are pairwise orthogonal non-zero tripotents in E, compare [8]. By [1] Lemma 7.8 we know that $\Theta_a = E_{00}(e_1)$ is a subtriple of E. Since Θ_a has triple rank $\widetilde{r}(\Theta_a) < n$ we get by induction hypothesis $r(\Theta_a) = \widetilde{r}(\Theta_a) \leq n - 1$, i.e. $r(E) \leq n = \widetilde{r}(E)$ by 2.2.ii. □

JB*-triples of finite rank can be characterized in many ways, compare also [8].

3.2 Proposition. *For every JB*-triple E the following conditions are equivalent.*

(i) *E has finite rank.*
(ii) *Every finite subset of E is contained in a subtriple of finite dimension.*
(iii) *Every $a \in \partial B$ has a (unique) representation $a = e + u$ with $u \in B$, e a tripotent and $e \perp u$.*
(iv) *For every $a \in E$ the operator $a \square a \in \mathcal{L}(E)$ is algebraic (i.e. satisfies a nontrivial polynomial equation).*
(v) *E is reflexive.*

JB*-triples E of finite rank behave essentially like those of finite dimension, compare [10] for the following discussion. A tuple $(e_1, \ldots, e_r)$ of pairwise orthogonal minimal tripotents in E is called a *frame in* E if $E_{00}(e_1,\ldots,e_r)=0$. All frames have the same length $r=r(E)$. The tripotent $\varepsilon(a):=e$ in 3.2.iv can be obtained by $e=\lim a^{2n+1}$. The fibres of the mapping $\varepsilon:\overline{D}\to M$ are the holomorphic arc components of $\overline{D}$, i.e. the smallest non-empty subsets $A\subset\overline{D}$ with the property: $f(\Delta)\subset A$ for every holomorphic mapping $f:\Delta\to\overline{D}$ with $f(\Delta)\cap A\neq\emptyset$. For every $a\in D$ and $e=\varepsilon(a)$ the holomorphic arc component of a is $\varepsilon^{-1}(e)=e+(D\cap E_{00}(e))$.

The n-dimensional Banach space $F=\ell^\infty_n$ is a JB*-triple with open unit ball Δ^n. Let $f_1=(1,0,\ldots,0),\ldots,f_n=(0,\ldots,0,1)$ be the standard basis of F. Suppose, E has finite rank and $L:F\to E$ is a linear isometry. Let $a_k:=L(f_k)$ and write $a_k=e_k+u_k$ with $e_k=\varepsilon(a_k)$ for all k. Then for all $j\neq k$ we have $a_j+\Delta a_k\subset\varepsilon^{-1}(a_j)=\varepsilon^{-1}(e_j)$. This implies $u_j+\Delta a_k\subset E_{00}(e_j)$ and hence $a_k\in E_{00}(e_j)$. The closed subtriple $E_{00}(e_j)$ contains with a_k also all odd powers of a_k and hence also the limit e_k, i.e. $e_j\perp e_k$ and $u_j\perp e_k$ for all $j\neq k$. This implies $u_j\perp e:=e_1+\ldots+e_n$ and also $n\le r(E)$. In case of equality all u_j vanish and $L(F)$ is a subtriple of E. This implies that then L is a triple homomorphism. Since for every $1\le k<n$ there exist linear isometries $\ell^\infty_k\to\ell^\infty_n$ that are not triple homomorphisms we have thus proved

3.3 Lemma. *For every JB*-triple E and every integer $r\ge 1$ the following conditions are equivalent.*

(i) *E has finite rank r.*

(ii) *$n\le r$ if there exists a linear isometry $\ell^\infty_n\to E$.*

(iii) *Every linear isometry $\ell^\infty_r\to E$ is a triple homomorphism.*

As a consequence, for every JB*-triple of finite rank, $r(E)$ is the maximal n such that there exists a linear isometry $\ell^\infty_n\to E$.

We are now ready to prove the main result of this section.

3.4 Theorem. *Let E, F be JB*-triples with $r(F)\le r(E)<\infty$ and open unit balls B, D. Suppose $f:B\to D$ is a holomorphic mapping such that the derivative $L:=df(0)\in\mathcal{L}(E,F)$ is an isometry. Then $r(F)=r(E)$, $f=L|B$ and L is a triple homomorphism. In particular, the rank function r on $\mathcal{JB}$ satisfies* Property A.

Proof. Fix $a\in E$ and put $r:=r(E)$. Then there exists a frame $(e_1,\ldots,e_r)$ in E and a spectral decomposition $a=\lambda_1e_1+\cdots+\lambda_re_r$ with coefficients $\lambda_i\ge 0$ for all i. Since L is an isometry $t\mapsto\sum_{i=1}^r t_iL(e_i)$ defines an isometry $R:\ell^\infty_r\to F$. From 3.3.ii we derive $r(E)=r(F)$ and also that R is a triple homomorphism. This implies $L(a)^3=L(a^3)$ for all $a\in E$, i.e. also L is a triple homomorphism. The set $\partial_eB\subset\partial B$ of all extreme boundary points of B coincides with $\{e\in M: E_{00}(e)=0\}$ and is a set of determinacy in E in the sense of [1]. Because of $L(\partial_eB)\subset\partial_eD$ we derive $f=L|B$ as a consequence of Corollary 3.3 in [1]. □

Suppose, E with open unit ball B is a JB*-triple of finite rank r. In [8] all equivalent norms Φ on E have been determined which are invariant under the group $\mathsf{GL}(B)\subset\mathsf{GL}(E)$. Among these are all p-norms for $1\le p\le\infty$ on E defined as follows: Write every $a\in E$ as linear combination $a=\lambda_1e_1+\lambda_2e_2+\cdots+\lambda_re_r$ for some frame $(e_1,e_2,\ldots,e_r)$ in E and put $\|a\|_p:=\|(\lambda_1,\lambda_2,\ldots,\lambda_r)\|_p$. Then $\partial_eB=\{a\in\overline{B}:\|a\|_p=r^{1/p}\}$, the original norm of E coincides with $\|\ \|_\infty$ and $\|\ \|_2$ is a Hilbert norm. In particular, E is isomorphic to a complex Hilbert space. Now suppose that F is another JB*-triple of finite rank and $L:E\to F$ is a linear map with $\|L\|\le 1$ and $\|L(a)\|_p=\|a\|_p$ for all $a\in E$ (i.e an isometry with respect to the p-norm on both spaces). Since $\overline{B}$ is the closed convex hull of ∂_eB in E it is clear that then L is also an isometry of JB*-triples. Thus as a consequence of our main Theorem ?? we get.

3.5 Proposition. *Let E,F with open unit balls B, D be JB*-triples of rank $r(F)\le r(E)<\infty$ and let $f:B\to D$ be a holomorphic mapping such that $L:=df0)$ is an isometry with respect to the p-norm for some $1\le p\le\infty$. Then $r(F)=r(E)$ and $f=L|B$ is linear.*

For $p=2$ and finite dimensions this result is already contained in [13].

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