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Peirce gradings and Peirce inner ideals in JBW*-triple factors

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Abstract. A Peirce inner ideal J in an anisotropic Jordan*-triple A gives rise to a Peirce grading (J_0, J_1, J_2) of A by defining

$$J_0 = J^{\perp}, \quad J_1 = \mathrm{Ker}(J) \cap \mathrm{Ker}(J^{\perp}), \quad J_2 = J,$$

where $J^{\perp}$ is the set of elements a of A for which $\{J\ a\ A\}$ is equal to $\{0\}$ and $\mathrm{Ker}(J)$ is the set of elements a of A for which $\{J\ a\ J\}$ is equal to $\{0\}$. It is shown that conversely, when A is a JBW*-triple factor, for each Peirce grading (J_0, J_1, J_2) of A such that both J_0 and J_2 are non-zero, both J_0 and J_2 are Peirce inner ideals the corresponding Peirce decompositions of A being given by

$$(J_0)_0 = J_2, (J_0)_1 = J_1, (J_0)_2 = J_0;$$
$$(J_2)_0 = J_0, (J_2)_1 = J_1, (J_2)_2 = J_2.$$

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1. Introduction. A *Peirce grading* (J_0, J_1, J_2) of a Jordan*-triple A consists of subspaces J_0, J_1 and J_2 of A such that

$$J_0 \oplus J_1 \oplus J_2 = A, \tag{1.1}$$

$$\{J_0\ J_2\ A\} = \{J_2\ J_0\ A\} = \{0\}, \tag{1.2}$$

and, for j, k, and l equal to 0, 1, or 2,

$$\{J_j\ J_k\ J_l\} \subseteq J_{j-k+l}, \tag{1.3}$$

if $j - k + l$ is equal to 0, 1 or 2, and

$$\{J_j\ J_k\ J_l\} = \{0\}, \tag{1.4}$$

otherwise. A study of Peirce gradings and involutive gradings of Jordan pairs and Jordan*-triples was carried out by Neher [27], one of his main conclusions being that, provided that the pair or triple in question was simple, semi-simple, and satisfied both the ascending and descending chain conditions on principal inner ideals, the two concepts were essentially the equivalent.

A complex Banach space A that is the dual of a Banach space A_* and the open unit ball in which is a bounded symmetric domain possesses an intrinsic triple product with respect to which it forms an anisotropic Jordan*-triple which is known as a JBW*-triple. The family of JBW*-triples includes that of JBW*-algebras which, in turn, includes that of W*-algebras, or von Neumann algebras. A JBW*-triple that does not contain a non-trivial weak*-closed triple ideal is said to be a *JBW*-triple factor*, examples of which are the six discrete Cartan factors considered by Cartan [5] in finite dimensions and by Kaup [23] in infinite dimensions, and the continuous factors studied by Horn and Neher [22]. A JBW*-triple factor need not be simple, nor need it satisfy the ascending or descending chain condition on principal inner ideals. However, it was shown in [18] that Neher's results can be extended to JBW*-triple factors, thereby providing an example of a phenomenon which often appears in the theory of Jordan structures in which a particular result that holds for a Jordan*-triple under strong algebraic conditions continues to hold when these conditions are replaced by the geometrical requirement that the Jordan*-triple is a JBW*-triple.

An inner ideal J in an anisotropic Jordan*-triple A is said to be complemented if

$$J \oplus \mathrm{Ker}(J) = A,$$

where

$$\mathrm{Ker}(J) = \{a \in A : \{J\ a\ J\} = \{0\}\}.$$

In the case in which J and its algebraic annihilator

$$J^{\perp} = \{a \in A : \{J\ a\ A\} = \{0\}\}$$

are complemented, the Jordan*-triple A enjoys the *generalized Peirce decomposition*

$$A = J_0 \oplus J_1 \oplus J_2 \tag{1.5}$$

corresponding to J, where

$$J_0 = J^{\perp}, \quad J_1 = \mathrm{Ker}(J) \cap \mathrm{Ker}(J^{\perp}), \quad J_2 = J. \tag{1.6}$$

In this case, for j equal to 0, 1, or 2, J_j is said to be the *Peirce j-space* corresponding to J. However, (J_0, J_1, J_2) is not, in general, a Peirce grading of A since, although conditions (1.1), (1.2), and (1.4) hold, condition (1.3) fails to hold when (j, k, l) is equal to $(0,1,1)$, $(1,1,0)$, $(1,0,1)$, $(1,1,1)$, $(2,1,1)$ $(1,1,2)$, $(1,2,1)$, $(2,1,0)$, and $(0,2,1)$. In the case in which all these Peirce relations hold J is said to be a *Peirce inner ideal.* If follows that every Peirce inner ideal J gives rise to a Peirce grading (J_0, J_1, J_2) given by (1.6). It was shown in [10, 12] that when A is a

JBW*-triple an inner ideal is complemented if and only if it is weak*-closed and the Peirce conditions in (1.3) in which (j,k,l) is equal to $(2,1,0)$ and $(0,1,2)$ are automatically satisfied. Examples do, however, exist in which the remaining seven conditions all fail to hold [14].

This note is devoted to showing that, for a Peirce grading (J_0, J_1, J_2) of a JBW*-triple factor A in which both J_0 and J_2 are non-zero, both J_0 and J_2 are Peirce inner ideals in A such that

$$(J_0)_0 = J_2, (J_0)_1 = J_1, (J_0)_2 = J_0, \quad (J_2)_0 = J_0, (J_2)_1 = J_1, (J_2)_2 = J_2,$$

thereby providing a converse to the result referred to above.

When A is a rectangular or a continuous hermitian factor every weak*-closed inner ideal J in A is Peirce, and, provided that $J^{\perp}$ is non-zero, $J^{\perp\perp}$ coincides with J, and the result proved is hardly surprising. On the other hand, when A is a spin triple or one of the two exceptional Cartan factors, most of the weak*-closed inner ideals in which are not Peirce, the result shows that such inner ideals cannot occur as the constituents J_0 and J_2 of a Peirce grading (J_0, J_1, J_2). The reader is referred to [11, 13, 14, 15, 16, 19] for details.

2. Preliminaries. A complex vector space A equipped with a triple product $(a,b,c) \mapsto \{a\ b\ c\}$ from $A \times A \times A$ to A which is symmetric and linear in the first and third variables, conjugate linear in the second variable and, for elements a, b, c and d in A, satisfies the identity

$$[D(a,b), D(c,d)] = D(\{a\ b\ c\}, d) - D(c, \{d\ a\ b\}), \tag{2.1}$$

where $[\ .\ ,\ .\ ]$ denotes the commutator, and D is the mapping from $A \times A$ to the algebra of linear operators on A defined by

$$D(a,b)c = \{a\ b\ c\},$$

is said to be a *Jordan*-triple*. A Jordan*-triple A for which the vanishing of $\{a\ a\ a\}$ implies that a itself vanishes is said to be *anisotropic*. A subspace J of a Jordan*-triple A such that $\{J\ J\ J\}$ is contained in J is said to be a *subtriple* of A. A subtriple J of A for which $\{J\ A\ J\}$ is contained in J is said to be an *inner ideal* of A and an inner ideal J in A for which both $\{A\ J\ A\}$ and $\{A\ A\ J\}$ are contained in J is said to be an *ideal* in A. For details of the properties of Jordan*-triples the reader is referred to [26].

A Jordan*-triple A which is also a dual Banach space such that D is norm-continuous from $A \times A$ to the Banach algebra $B(A)$ of bounded linear operators on A, and, for each element a in A, $D(a,a)$ is positive in the sense of [3] and satisfies the condition that

$$\|D(a,a)\| = \|a\|^2,$$

is said to be a *JBW*-triple*. A complex dual Banach space possesses a triple product with respect to which it forms a JBW*-triple if and only if its open unit ball is a bounded symmetric domain. The predual A_* of a JBW*-triple is unique and the triple product is separately weak*-continuous. Every subtriple of a JBW*-triple is an anisotropic Jordan*-triple, and a weak*-closed subtriple of a JBW*-triple is a JBW*-triple. A norm-closed subspace J of a JBW*-triple A is an ideal if and only if $\{J\,J\,A\}$ is contained in J. For details of these and related results the reader is referred to [1, 2, 4, 6, 7, 20, 21, 24, 25, 28, 29].

A pair a and b of elements of the JBW*-triple A is said to be *orthogonal* if $D(a,b)$ is equal to zero. For a non-empty subset L of A, the subset $L^\perp$ consisting of elements of A orthogonal to all elements of L is a weak*-closed inner ideal in A, known as the (algebraic) *annihilator* of L, and the weak*-closed subspace $\mathrm{Ker}(L)$ consisting of elements a of A for which $\{L\ a\ L\}$ is equal to $\{0\}$ is known as the *kernel* of L. Clearly, $L^\perp$ is contained in $\mathrm{Ker}(L)$ and $L \cap \mathrm{Ker}(L)$ is contained in $\{0\}$. A subtriple J of A is said to be *complemented* if

$$J \oplus \mathrm{Ker}(J) = A.$$

The results of [10, 12] show that a subtriple is complemented if and only if it is a weak*-closed inner ideal.

Let $\mathcal{I}(A)$ be the complete lattice of weak*-closed inner ideals in the JBW*-triple A. For each element J in $\mathcal{I}(A)$, the annihilator $J^\perp$ also lies in $\mathcal{I}(A)$, and A enjoys the generalized Peirce decomposition described in (1.5)–(1.6). Two elements J and K of $\mathcal{I}(A)$ with Peirce spaces J_0, J_1, and J_2 and K_0, K_1, and K_2 are said to be *compatible* if

$$A = \bigoplus_{j,k=0,1,2} J_j \cap K_k.$$

An element I of $\mathcal{I}(A)$ is said to be *central* if I is compatible with every element J in $\mathcal{I}(A)$. An element I is central if and only if I is an ideal in A or, equivalently, if the Peirce 1-space I_1 corresponding to I is equal to $\{0\}$. The family $\mathcal{ZI}(A)$ of ideals in $\mathcal{I}(A)$ forms a Boolean sub-complete lattice of $\mathcal{I}(A)$. For details, the reader is referred to [8, 9].

3. Main Result. In this section the main result is proved. Its proof depends upon a series of lemmas the background to which must first be presented.

A pair (B_+, B_-) of subtriples of a Jordan*-triple A is said to be an *involutive grading* of A if

$$A = B_+ \oplus B_-,$$

$$\{B_+\ B_-\ B_+\} \subseteq B_-, \qquad \{B_-\ B_+\ B_-\} \subseteq B_+, \tag{3.1}$$

$$\{B_+\ B_+\ B_-\} \subseteq B_-, \qquad \{B_-\ B_-\ B_+\} \subseteq B_+. \tag{3.2}$$

Observe that, by symmetry, if (B_+, B_-) is an involutive grading then so also is (B_-, B_+) which is said to be the grading *opposite* to (B_+, B_-). A linear mapping ϕ from A to itself, which is a triple automorphism of A and satisfies the condition that ϕ^2 coincides with the identity operator id_A on A, is said to be an *involutive automorphism* of A. Observe that, if ϕ is an involutive automorphism of A then so also is the mapping $-\phi$ defined, for each element a in A, by

$$(-\phi)a = -\phi a.$$

For each involutive automorphism ϕ of A, let

$$B_+^\phi = \{a \in A : \phi a = a\} \qquad B_-^\phi = \{a \in A : \phi a = -a\}.$$

Then, (B_+^ϕ, B_-^ϕ) is an involutive grading and the mapping $\phi \mapsto (B_+^\phi, B_-^\phi)$ is a bijection from the set of involutive automorphisms of A onto the set of involutive gradings of A, such that $(B_+^{-\phi}, B_-^{-\phi})$ coincides with (B_-^ϕ, B_+^ϕ). Observe that the linear mappings T_ϕ and $T_{-\phi}$ defined by

$$T_\phi = \tfrac{1}{2}(\mathrm{id}_A + \phi), \quad T_{-\phi} = \tfrac{1}{2}(\mathrm{id}_A - \phi)$$

are linear projections onto the subtriples B_+^ϕ and B_-^ϕ, respectively. The following lemma describes how Peirce gradings give rise to involutive gradings.

Lemma 3.1. *Let A be a Jordan*-triple, let (J_0, J_1, J_2) be a Peirce grading of A, and let P_0, P_1, and P_2 be the linear projections onto the subspaces J_0, J_1, and J_2, respectively. Then, $(J_0 \oplus J_2, J_1)$ is an involutive grading of A, the corresponding involutive automorphism ϕ being given by*

$$\phi = 2P_0 + 2P_2 - \mathrm{id}_A = \mathrm{id}_A - 2P_1 = P_0 - P_1 + P_2,$$

and the corresponding projections T_ϕ and $T_{-\phi}$ being given by

$$T_\phi = P_0 + P_2, \quad T_{-\phi} = \mathrm{id}_A - T_\phi = P_1.$$

Proof. See [18, Lemma 4.1]. □

When A is a JBW*-triple, the results of [1, 2, 21, 24, 25] show that every involutive automorphism ϕ of A is automatically a weak*-continuous isometry. It follows that, for any involutive grading (B_+^ϕ, B_-^ϕ), the subtriples B_+^ϕ and B_-^ϕ of A are weak*-closed and the corresponding projections T_ϕ and $T_{-\phi}$ are weak*-continuous and contractive.

In the next result the general properties of Peirce gradings in JBW*-triples are described.

Lemma 3.2. *Let A be a JBW*-triple and let (J_0, J_1, J_2) be a Peirce grading of A. Then, the following results hold.*

(i) *The subspaces J_0 and J_2 are weak*-closed inner ideals in A, and J_1 and $J_0 \oplus J_2$ are weak*-closed subtriples of A.*

(ii) *The weak***-closed inner ideals* J_0 *and* J_2 *are weak***-closed ideals in the JBW***-triple* $J_0 \oplus J_2$.
(iii) *The weak***-closed inner ideals* J_0 *and* J_2 *satisfy:*

$$\mathrm{Ker}(J_0) = J_1 \oplus J_2, \qquad \mathrm{Ker}(J_2) = J_1 \oplus J_0,$$

and

$$J_1 = \mathrm{Ker}(J_0) \cap \mathrm{Ker}(J_2).$$

(iv) *The Peirce spaces* $(J_0)_0$, $(J_0)_1$, *and* $(J_0)_2$ *and* $(J_2)_0$, $(J_2)_1$, *and* $(J_2)_2$ *corresponding to the weak***-closed inner ideals* J_0 *and* J_2 *satisfy:*

$$(J_0)_0 = J_2 \oplus (J_0)_0 \cap J_1, \quad (J_0)_1 \oplus (J_0)_0 \cap J_1 = J_1, \quad (J_0)_2 = J_0;$$

$$(J_2)_0 = J_0 \oplus (J_2)_0 \cap J_1, \quad (J_2)_1 \oplus (J_2)_0 \cap J_1 = J_1, \quad (J_2)_2 = J_2.$$

Proof. See [18, Lemmas 4.2, 4.3, and 4.5]. □

The following result is a strengthened version of the main result of [18].

Lemma 3.3. *Let* A *be a JBW***-triple factor and let* (B_+, B_-) *be an involutive grading of* A. *Then, either, there exist non-zero weak***-closed ideals* J_0 *and* J_2 *in* B_+ *such that*

$$J_0 = (J_2)^\perp \cap B_+, \quad J_2 = (J_0)^\perp \cap B_+,$$

in which case, writing J_1 *equal to* B_-, (J_0, J_1, J_2) *is a Peirce grading of* A *and* J_0 *and* J_2 *are JBW***-triple factors, uniquely defined up to the exchange of* J_0 *and* J_2, *or* B_+ *is a JBW***-triple factor.*

Proof. All except for the uniqueness of the pair J_0 and J_2 of non-zero weak*-closed ideals in B_+ was proved in [18, Theorem 5.5]. Let I be a non-zero, weak *-closed ideal in the JBW*-triple B_+ with I not equal to B_+. Then,

$$\{I \cap J_0 \ I \cap J_0 \ J_0\} \subseteq I \cap J_0$$

and, by [4], $I \cap J_0$ is a weak*-closed ideal in the JBW*-triple factor J_0. It follows that $I \cap J_0$ is equal to either $\{0\}$ or J_0. Similarly, $I \cap J_2$ is equal to either $\{0\}$ or J_2. However, since I, J_0 and J_2 are compatible in the JBW*-triple B_+,

$$I = (I \cap J_0) \oplus (I \cap J_2). \tag{3.3}$$

If $I \cap J_0$ and $I \cap J_2$ are both equal to $\{0\}$ then, by (3.3), I is equal to $\{0\}$, giving a contradiction. If $I \cap J_0$ is equal to J_0 and $I \cap J_2$ is equal to J_2 then, by (3.3), I is equal to B_+, giving a contradiction. If $I \cap J_0$ is equal to J_0 and $I \cap J_2$ is equal to $\{0\}$ then, by (3.3), I is equal to J_0. Similarly, if $I \cap J_0$ is equal to $\{0\}$ and $I \cap J_2$ is equal to J_2 then, by (3.3), I is equal to J_2. This completes the proof of the lemma. □

By symmetry, the result above also holds with the rôles of B_+ and B_- interchanged. Before proceeding to the statement and proof of the main result, one further property of involutive gradings is required.

Lemma 3.4. *Let A be a JBW*-triple and let (B_+, B_-) be an involutive grading of A. Then the kernel* $\mathrm{Ker}(B_+)$ *of the weak*-closed subtriple B_+ of A is a weak*-closed ideal in the JBW*-triple B_-.*

Proof. By [12, Lemma 4.2], $\mathrm{Ker}(B_+)$ is a weak*-closed subspace of A. Suppose that a is an element of $\mathrm{Ker}(B_+)$, and let a_+ and a_- be the unique elements of B_+ and B_-, respectively, such that

$$a = a_+ + a_-.$$

Then,

$$0 = \{a_+ \ a \ a_+\} = \{a_+ \ a_+ \ a_+\} + \{a_+ \ a_- \ a_+\}.$$

Since B_+ is a subtriple, $\{a_+ \ a_+ \ a_+\}$ lies in B_+ and, by (3.1), $\{a_+ \ a_- \ a_+\}$ lies in B_-. It follows that both elements are equal to zero, and, by anisotropy, a_+ is equal to zero. Hence, a is equal to a_- and, therefore, lies in B_-. It follows that $\mathrm{Ker}(B_+)$ is contained in B_-. By [4], it remains to show that

$$\{\mathrm{Ker}(B_+) \ \mathrm{Ker}(B_+) \ B_-\} \subseteq \mathrm{Ker}(B_+). \tag{3.4}$$

Let a_+ and c_+ be elements of B_+, let d_- and e_- be elements of $\mathrm{Ker}(B_+)$, and let b_- be an element of B_-. Then, using (2.1) and (3.2),

$$\begin{aligned}
\{a_+ \ \{d_- \ e_- \ b_-\} \ c_+\} &= D(a_+, \{d_- \ e_- \ b_-\})c_+ \\
&= D(\{e_- \ b_- \ a_+\}, d_-)c_+ + D(a_+, d_-)D(e_-, b_-)c_+ \\
&\quad - D(e_-, b_-)D(a_+, d_-)c_+ \\
&\in D(B_+, \mathrm{Ker}(B_+))B_+ + D(B_+, \mathrm{Ker}(B_+))B_+ \\
&\quad - D(\mathrm{Ker}(B_+), B_-)\{B_+ \ \mathrm{Ker}(B_+) \ B_+\} \\
&= \{0\},
\end{aligned}$$

thereby completing the proof of (3.4). □

It is now possible to prove the main result of the paper.

Theorem 3.5. *Let A be a JBW*-triple factor and let (J_0, J_1, J_2) be a Peirce grading of A for which both J_0 and J_2 are non-zero. Then, J_0 and J_2 are Peirce weak*-closed inner ideals in A with Peirce spaces given by*

$$(J_0)_0 = J_2, \quad (J_0)_1 = J_1, \quad (J_0)_2 = J_0; \tag{3.5}$$

$$(J_2)_0 = J_0, \quad (J_2)_1 = J_1, \quad (J_2)_2 = J_2. \tag{3.6}$$

Proof. If J_1 is equal to $\{0\}$ then, by Lemma 3.2(ii), J_0 and J_2 are non-zero orthogonal weak*-closed ideals in A with direct sum equal to A contradicting the condition that A is a JBW*-triple factor.

Therefore, it can be assumed that J_0, J_1, and J_2 are all non-zero. Observe that, by symmetry, since (J_0, J_1, J_2) is a Peirce grading, so also is (J_2, J_1, J_0). Moreover, also by symmetry, it is sufficient to prove the result for J_0. By Lemma 3.2(iv), in order to show that (3.5) holds, it is sufficient to show that the subtriple $(J_0)_0 \cap J_1$ of A is equal to $\{0\}$. Since (J_2, J_1, J_0) is a Peirce grading it will then follow from (1.2)–(1.4) that J_0 is a Peirce weak*-closed inner ideal.

Let a_1 be an element of $(J_0)_0 \cap J_1$, let b_0 and c_0 be elements of J_0, and let b_2 and c_2 be elements of J_2. Using (1.2)–(1.4) and the orthogonality of a_1 with both b_0 and c_0, it can be seen that

$$\{b_0 + b_2\ a_1\ c_0 + c_2\} = \{b_0\ a_1\ c_0\} + \{b_0\ a_1\ c_2\} + \{b_2\ a_1\ c_0\} + \{b_2\ a_1\ c_2\} = 0.$$

It follows that the subtriple $(J_0)_0 \cap J_1$ is contained in the kernel $\mathrm{Ker}(J_0 \oplus J_2)$ of the subtriple $J_0 \oplus J_2$ of A. The proof will, therefore, be complete if it can be shown that $\mathrm{Ker}(J_0 \oplus J_2)$ is equal to $\{0\}$.

It follows from Lemma 3.1 that $(J_0 \oplus J_2, J_1)$ is an involutive grading of A, and, therefore, using Lemma 3.4, $\mathrm{Ker}(J_0 \oplus J_2)$ is a weak*-closed ideal in the JBW*-triple J_1. Applying Lemma 3.3 to the involutive grading $(J_1, J_0 \oplus J_2)$ of A it follows that three possibilities arise. These are:

(i) $\mathrm{Ker}(J_0 \oplus J_2) = K_0$, $\mathrm{Ker}(J_0 \oplus J_2)^{\perp} \cap J_1 = K_2$,
where K_0 and K_2 are non-zero weak*-closed ideals in the JBW*-triple J_1 with direct sum J_1 and $(K_0, J_0 \oplus J_2, K_2)$ is a Peirce grading of A;
(ii) $\mathrm{Ker}(J_0 \oplus J_2) = J_1$;
(iii) $\mathrm{Ker}(J_0 \oplus J_2) = \{0\}$.

Suppose that (i) holds. Let a be an element of $J_0 \oplus J_2$ and let b be an element of K_2. Then, applying (1.3) to the Peirce grading $(K_0, J_0 \oplus J_2, K_2)$ of A,

$$\{a\ b\ a\} \in \{J_0 \oplus J_2\ K_2\ J_0 \oplus J_2\} \subseteq K_0 = \mathrm{Ker}(J_0 \oplus J_2). \tag{3.7}$$

It follows from (3.7) that

$$\{a\ \{a\ b\ a\}\ a\} \in \{J_0 \oplus J_2\ \mathrm{Ker}(J_0 \oplus J_2)\ J_0 \oplus J_2\} = \{0\}. \tag{3.8}$$

Using the standard Peirce quadratic relation, that can be proved using (2.1) and (3.8),

$$\{\{a\ b\ a\}\ \{a\ b\ a\}\ \{a\ b\ a\}\} = \{a\{b\{a\ \{a\ b\ a\}\ a\}b\}a\} = 0.$$

By anisotropy, it follows that the element $\{a\ b\ a\}$ is equal to zero. Using the linearity of the triple product it can be seen that

$$\{J_0 \oplus J_2\ K_2\ J_0 \oplus J_2\} = \{0\},$$

and, hence, that

$$K_2 \subseteq \mathrm{Ker}(J_0 \oplus J_2) = K_0,$$

yielding a contradiction.

Now, suppose that (ii) holds. Then, it follows from (1.1) that

$$J_0 \oplus J_2 \oplus \mathrm{Ker}(J_0 \oplus J_2) = J_0 \oplus J_1 \oplus J_2 = A,$$

and, therefore, that $J_0 \oplus J_2$ is a complemented subtriple of A. By [12, Lemma 4.1], it can be seen that $J_0 \oplus J_2$ is a weak*-closed inner ideal in A. In this case, J_0 is a weak*-closed ideal in the weak*-closed inner ideal $J_0 \oplus J_2$ in A, and, by [17, Corollary 3.6], there exists a weak*-closed ideal I in A such that

$$J_0 = I \cap (J_0 \oplus J_2). \tag{3.9}$$

Since A is a JBW*-triple factor, I is equal either to $\{0\}$, in which case J_0 is equal to $\{0\}$, or to A, in which case J_2 is equal to $\{0\}$, both of which lead to a contradiction.

It therefore follows that (iii) holds and the proof of the theorem is complete. □

A consequence of this result is that Lemma 3.3 can be further strengthened.

Corollary 3.6. *Let A be a JBW*-triple factor and let (B_+, B_-) be an involutive grading of A. Then, either there exists a non-zero Peirce weak*-closed inner ideal J_0 in A with Peirce spaces $(J_0)_0$, $(J_0)_1$, and $(J_0)_2$ such that $(J_0)_0$ is non-zero, $(J_0)_0$ and $(J_0)_2$ are JBW*-triple factors, and*

$$B_+ = (J_0)_0 \oplus (J_0)_2, \quad B_- = (J_0)_1,$$

or the weak-closed subtriple B_+ of A is a JBW*-triple factor. The decomposition above is unique up to the exchange of $(J_0)_0$ and $(J_0)_2$,*

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