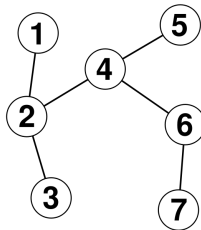


Name:

SAMPLE EXAM #1

1. COUNTING TREES

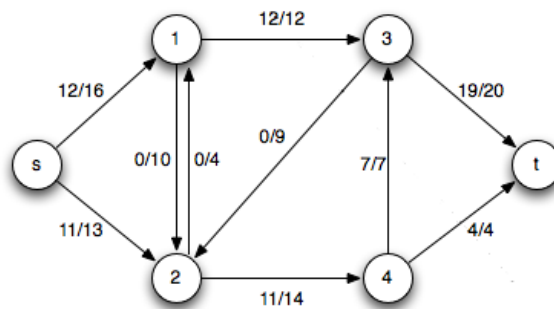
a) Give the Prüfer code of the following labeled tree:



- b) Finish the following sentence (without a proof): For any fixed n , the Prüfer encoding establishes a bijection between the set of labeled trees on vertex set $\{1, \dots, n\}$ and the set \dots
- c) State Cayley's theorem about the number of labeled trees.

2. NETWORK FLOWS

- a) State the maximum flow – minimum cut theorem.
- b) Consider the network (and flow) given in the figure. The capacity of an edge is the second number on it.

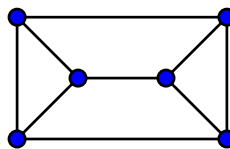


Prove that the given flow f is a maximum flow, by finding a minimum cut, which proves the maximality of f . Explain briefly why the maximality of f follows. (You can use Ford–Fulkerson-algorithm to find such a cut, but you do not have to demonstrate how you found the cut.)

- c) What we call augmenting path? Does there exist an augmenting path with respect to a maximum flow (for example, with respect to the flow in the figure)? Justify your answer.

3. MATCHINGS

a) Find a perfect matching in the following graph:



- b) What do we mean on König set? State the marriage theorem (about perfect matching in bipartite graphs), and prove its easy direction.
- c) What do we call augmenting path? Why does the existence of an augmenting path imply that the current matching is not of maximum size?