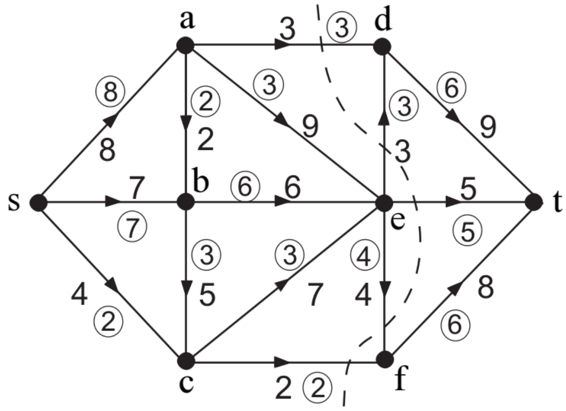
## 4. Network flows

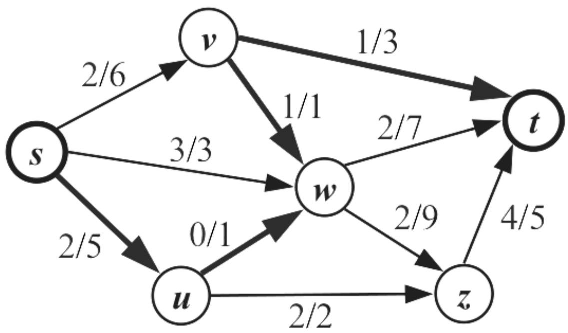
**1.** Consider the network in the figure ($s$ is the source, $t$ is the sink; and the uncircled numbers denote edge capacities).

a) Check that the circled numbers define a feasible flow.
b) Determine the capacity of the $[S,T]$-cut for $S = \{s,a,b,c,e\}$ and $T = \{d,f,t\}$.
c) Determine the value of the given flow.
d) Is this flow value maximal? Justify your answer.

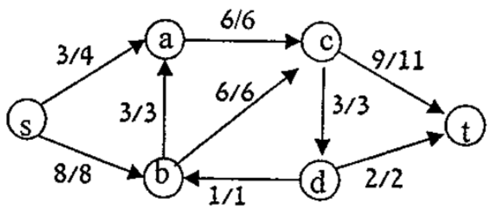


**2.** Consider the network in the figure (using the conventional notations).

a) Check that the first numbers on edges determine a feasible flow.
b) Check that the path $suwvt$ is an augmenting path, and using that, find a feasible flow with greater flow value.
c) Find a maximum flow in the network, and prove its maximality.



**3.** Consider the network in the figure (using the conventional notations).



a) Check that the first numbers on edges determine a feasible flow and determine the value of the given flow.
b) Find an augmenting path, and augment the flow along it.
c) Is the obtained flow is a maximum flow? (Justify your answer.)

**4.** Consider the network in the figure (using the conventional notations).

a) Using the Ford–Fulkerson algorithm, find an augmenting path, and augment the flow along it.

b) Repeat the above procedure until you find a maximum flow. Also provide an $[S, T]$-cut that proves the optimality of the flow.