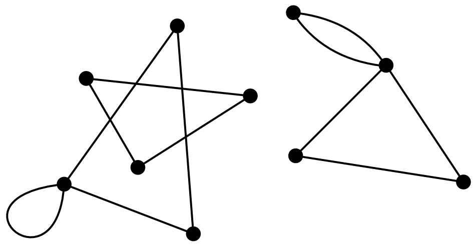
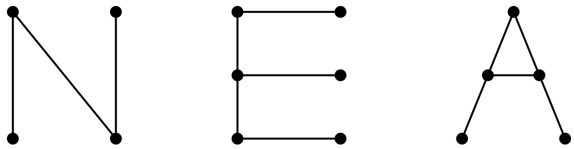
2. Connectivity. Trees

1. Determine whether the following multigraph is connected or not. (If not, give the number of components.)



2. Which of the following graphs are isomorhic to their complement?



G_1 G_2 G_3

3. G is a graph with degree sequence $1, 1, 1, 2, 2, 2, 5$. How many edges does $\overline{G}$ have? ($\overline{G}$ denotes the complement of G.)

4. In a graph G precisely two vertices have odd degree. Prove that there exists a path between these vertices in G.

5. Prove that G or $\overline{G}$ is connected, for any graph G.

6. Prove that if a simple graph G has $2n$ vertices and every vertex of G has degree at least n, then G is connected.

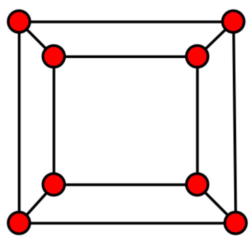
7. Prove that in a connected graph two longest paths always have a common vertex.

8. Prove that if in a graph G every vertex has degree at least 2, then G contains a cycle.

9.$^+$ Prove that if in a graph G every vertex has degree at least 3, then G contains an even cycle.

10. What happens if we add a new edge to a tree?

11. Is it possible to find two edge-disjoint spanning trees in the cube graph?



12. Show that in a connected graph G with at least 2 vertices, there always exists a vertex whose removal does not disconnect G.