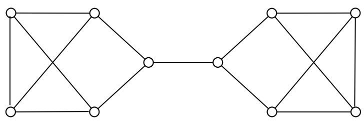
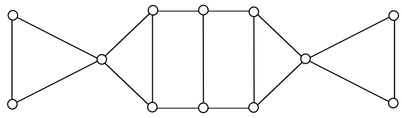
6. Application of network flows

1. Find the largest numbers k and l such that following graph is k-connected and l-edge-connected:

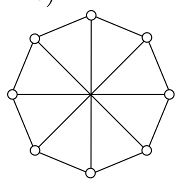
a)



b)



c)



d) Petersen graph;

e) the complete bipartite graph $K_{m,n}$;

2. a) Prove that every k-connected graph is also k-edge-connected.

b) Construct a graph which is 2022-edge-connected, but not 2-connected.

3. In a k-connected graph G, let $s, t_1, t_2, \ldots, t_k$ be $k+1$ distinct vertices of G, arbitrarily given. Prove that there exist k vertex-disjoint paths (apart from the initial vertex) in G such that the i'th path goes from s to t_i, for all $i \in \{1, \ldots, k\}$.

4.$^+$ Prove that in k-connected graph G (where $k \geq 2$), for any k vertices of G there exists a cycle containing them.

5. A large university with $3k$ academic departments must appoint an important committee. One professor will be chosen from each department. Some professors have joint appointments in two or more departments, but each must be the designated representative of at most one department. We must use k assistant professors, k associate professors, and k full professors (each professor has only one rank). How can the committee be found?

6. Project selection problem. Consider the following set of projects:

Name	Revenue	Pre-requisites
A	6	D
B	9	D
C	-8	
D	-12	
E	7	C, D

Find a feasible set of projects with maximal revenue.