## 4. Enumeration of spanning trees

1. Give the number of spanning trees of the graph
a) $P_{n}$, the path with $n$ edges;
b) $C_{n}$, the cycle on $n$ vertices.
2. How many spanning trees does the following graph have?

3. Among the spanning trees of $K_{n}$, how many stars and paths are?
4. Count the number of spanning of $K_{n}$ in which the fixed vertex $u$ is a leaf. (Here $n \geq 2$.)
5. Determine the Prüfer code of the following labeled tree.

6. a) Find the labeled tree whose Prüfer code is 5, 3, 3, 3, 1, 4 .
b) Find the labeled tree whose Prüfer code is $1,5,1,5,9,8,2$.
7. Deduce Cayley's theorem from Kirchhoff's matrix tree theorem.
8. We leave an edge from the complete graph on $n \geq 3$ vertices. How many spanning trees does the obtained graph have?
9. Count the number of spanning of the complete biparite graph $K_{m, n}$.
