## 2. Connectivity

1. Determine whether the following multigraph is connected or not. (If not, give the number of components.)

2. Which of the following graphs are isomorhic to their complement?

3. $G$ is a graph with degree sequence $1,1,1,2,2,2,5$. How many edges does $\bar{G}$ have? $(\bar{G}$ denotes the complement of $G$.)
4. In a graph $G$ precisely two vertices have odd degree. Prove that there exists a path between these vertices in $G$.
5. Prove that $G$ or $\bar{G}$ is connected, for any graph $G$.
6. Prove that if $G$ has $2 n$ vertices and every vertex of $G$ has degree at degree at least $n$, then $G$ is connected.
7. Prove that in a connected graph two longest paths always have a common vertex.
8. Prove that if in a graph $G$ every vertex has degree at least 2 , then $G$ contains a cycle.
9.+ Prove that if in a graph $G$ every vertex has degree at least 3 , then $G$ contains an even cycle.
