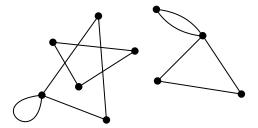
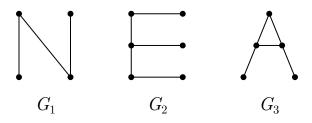
## 2. Connectivity

**1.** Determine whether the following multigraph is connected or not. (If not, give the number of components.)



2. Which of the following graphs are isomorphic to their complement?



**3.** G is a graph with degree sequence 1, 1, 1, 2, 2, 2, 5. How many edges does  $\overline{G}$  have? ( $\overline{G}$  denotes the complement of G.)

**4.** In a graph G precisely two vertices have odd degree. Prove that there exists a path between these vertices in G.

**5.** Prove that G or  $\overline{G}$  is connected, for any graph G.

**6.** Prove that if G has 2n vertices and every vertex of G has degree at degree at least n, then G is connected.

7. Prove that in a connected graph two longest paths always have a common vertex.

8. Prove that if in a graph G every vertex has degree at least 2, then G contains a cycle.

**9.**<sup>+</sup> Prove that if in a graph G every vertex has degree at least 3, then G contains an even cycle.