

1. BASICS

1. In a group of 8 people, every person writes down how many friends he has in this group. They wrote the following numbers: 3, 3, 3, 3, 4, 4, 4, 5. Show that at least one number is wrong. (The friendships are mutual.)
2. In a group of 8 people, every person writes down how many friends he has in this group. They wrote the following numbers: 0, 1, 2, 3, 4, 5, 6, 7. Show that at least one number is wrong. (The friendships are mutual.)
3. Does there exist a multigraph with degrees
 - a) 9, 7, 6, 6, 5, 4, 3, 3, 3, 1
 - b) 8, 7, 6, 6, 5, 4, 3, 3, 3, 1?
4. How many simple graphs have degree sequence 7, 7, 7, 7, 5, 5, 4, 4 (up to isomorphism)?
5. In a chess competition with 10 participants 11 games have been already played. Prove that there exists a participant who played at least 3 games.
6. After a party, every participant tells us how many people (of opposite sex) he or she danced with. We get the following numbers: 9, 9, 9, 9, 6, 6, 6, 5, 3, 3, 3, 3, 3, 3, 3. Prove that at least one of them is wrong.
7. Prove that there exists a k -regular graph with n vertices exists if and only if kn is even and $k \leq n - 1$.
8. In a chess competition involving 10 girls and 20 boys, every girl played exactly 6 games. We know that there were exactly 34 games in which a boy and a girl played against each other. What is the number of “girl vs. girl” games?
9. In a group of 9 people, every person gives 100\$ to exactly 5 other people. Prove that there exist two people whose money changed by the same amount.