Sample exam #1
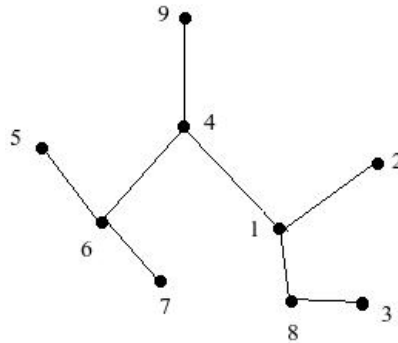
1. Counting trees

a) State Cayley's theorem about the number of labeled trees.

b) Draw all labeled trees on 3 vertices (with labels 1, 2, 3), and compare their number with Cayley's theorem.

c) Give the Prüfer code of the following labeled tree:

2. k-connectivity

a) When do we call a graph k-edge-connected, or k-connected?

b) What is the connection between these two notions?

c) State Menger's theorems (equivalent descriptions of k-edge-connectivity and k-connectivity).

3. Edge coloring

a) What do we mean on proper edge coloring and edge chromatic number of a graph?

b) State Vizing's theorem.

c) Give a proper edge coloring of the following graph with 3 colors:
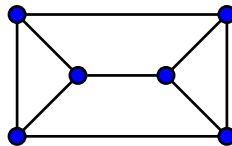
d) What do we know about the edge chromatic number of *bipartite* graphs? Prove your statement for *regular* bipartite graphs.

4. Chinese postman problem

a) What is the chinese postman problem?

b) Consider the following problem (which is the part of the algorithm solving chinese postman problem): There is a graph G given, and for each edge e of G, a natural number w_e is also given. How can we decide if there exists a closed walk in G which visits every edge e of G exactly w_e times? Justify your answer.