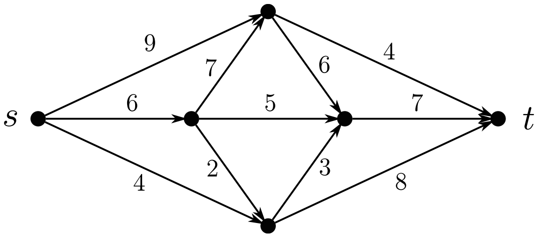
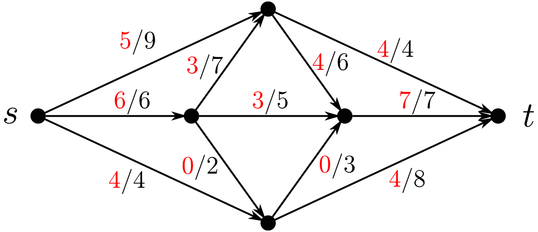
(G, s, t, c)



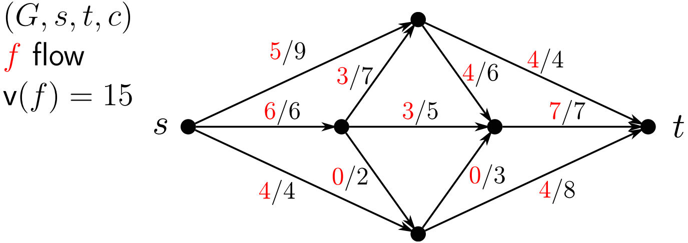
Problem. Find a maximum-value flow in the following network.

(G, s, t, c)
f flow
$\mathsf{v}(f) = 15$

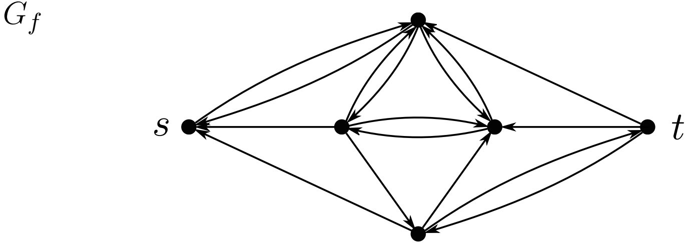


Problem. Find a maximum-value flow in the following network.

Solution. The starting point of the Ford–Fulkerson-algorithm is an arbitrary feasible flow. In practice, we can always choose the everywhere-zero flow, but in this presentation our starting point is a less trivial feasible flow.



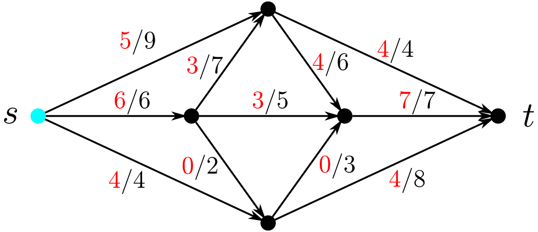
Searching for augmenting path



(G, s, t, c)

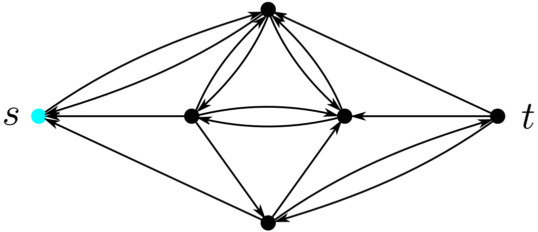
f flow

$\mathsf{v}(f) = 15$



Searching for augmenting path

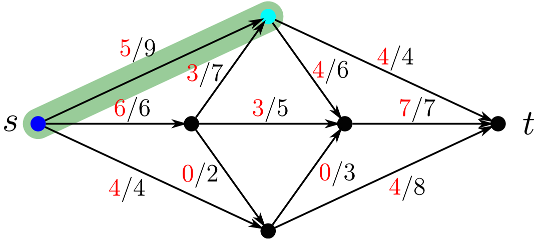
G_f



(G, s, t, c)

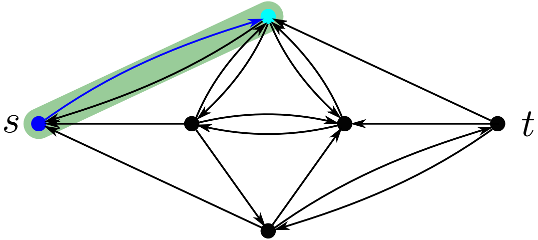
f flow

$\mathsf{v}(f) = 15$



Searching for augmenting path

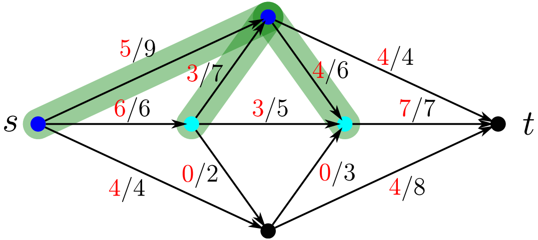
G_f



(G, s, t, c)

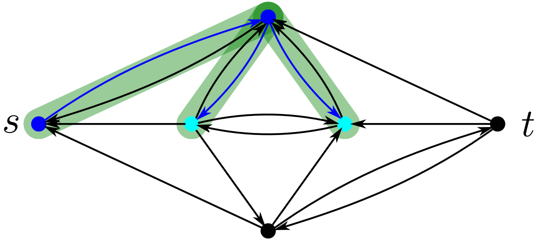
f flow

$\mathsf{v}(f) = 15$



Searching for augmenting path

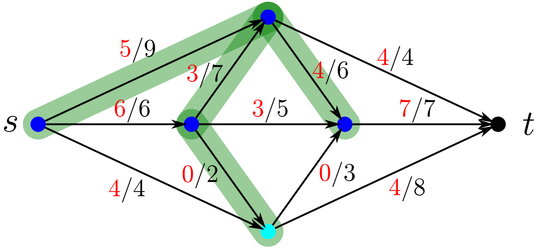
G_f



(G, s, t, c)

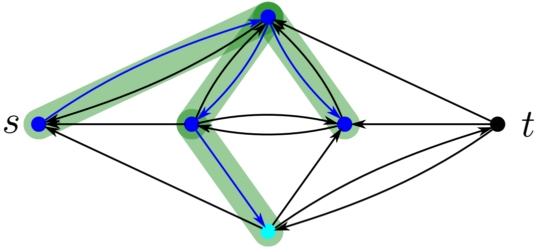
f flow

$\mathsf{v}(f) = 15$



Searching for augmenting path

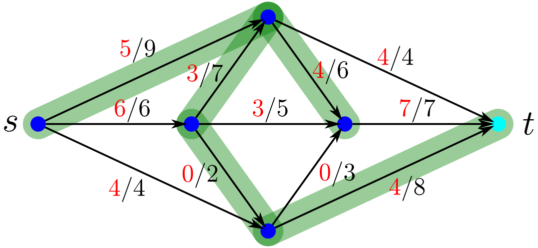
G_f



(G, s, t, c)

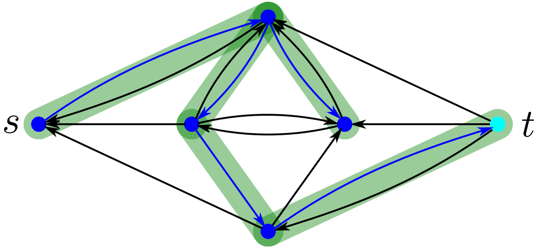
f flow

$\mathsf{v}(f) = 15$

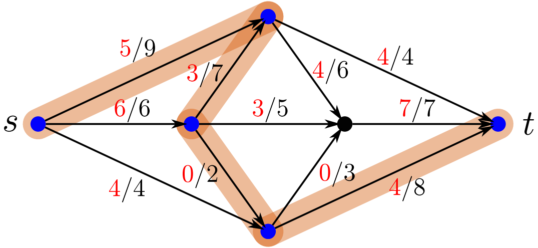


Searching for augmenting path

G_f

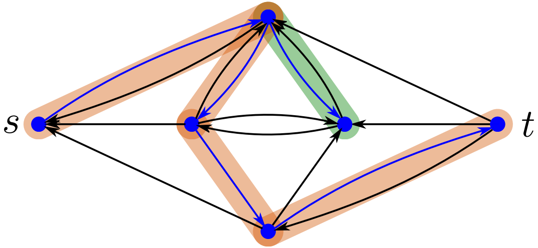


(G, s, t, c)
f flow
$\mathsf{v}(f) = 15$



An augmenting path was found.

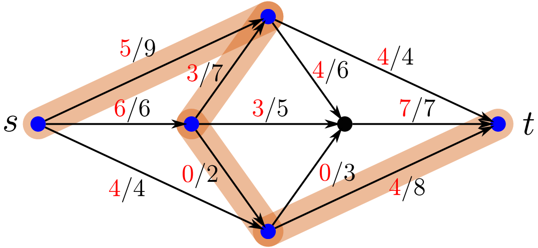
G_f



(G, s, t, c)

f flow

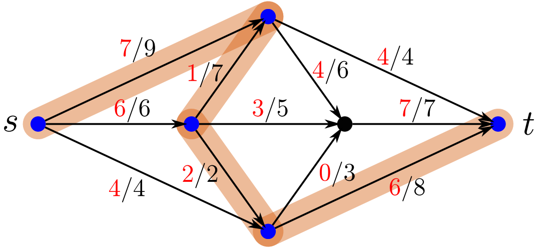
$\mathsf{v}(f) = 15$



Augmentation: $\delta = \min\{4, 3, 2, 4\} = 2$

G_f

(G, s, t, c)
f flow
$\mathsf{v}(f) = 17$



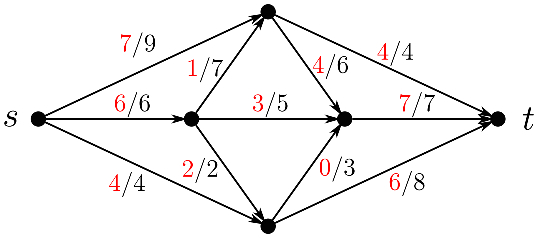
Augmentation: $\delta = \min\{4, 3, 2, 4\} = 2$

G_f

(G, s, t, c)

f flow

$\mathsf{v}(f) = 17$

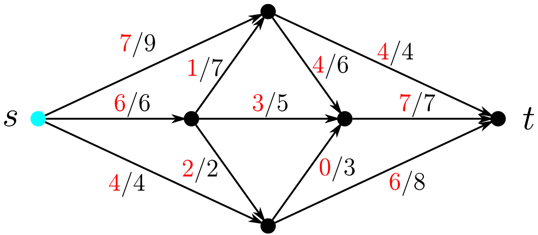


G_f

(G, s, t, c)

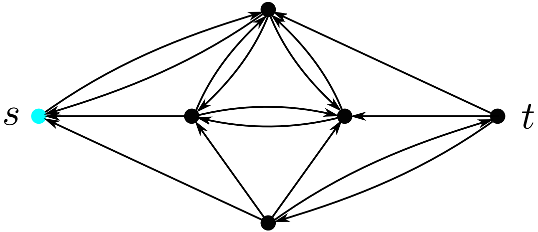
f flow

$\mathsf{v}(f) = 17$



Searching for augmenting path

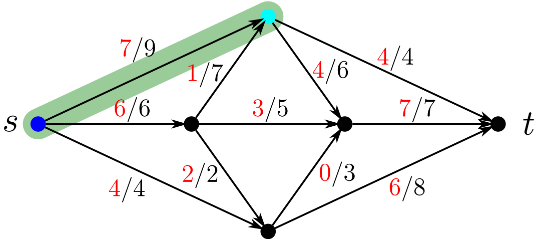
G_f



(G, s, t, c)

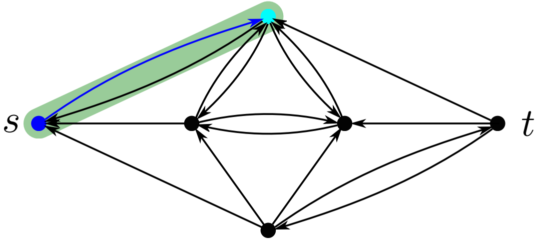
f flow

$\mathsf{v}(f) = 17$



Searching for augmenting path

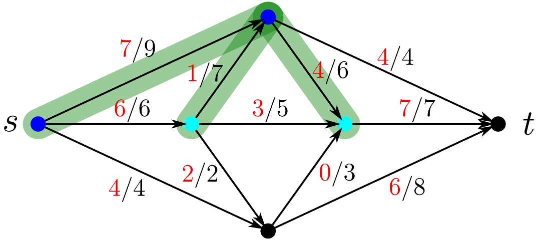
G_f



(G, s, t, c)

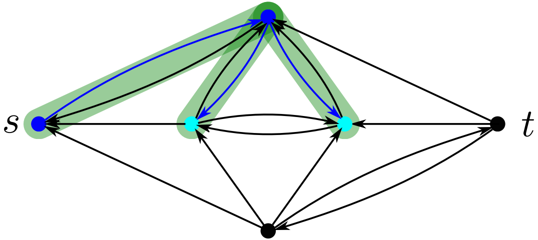
f flow

$\mathsf{v}(f) = 17$



Searching for augmenting path

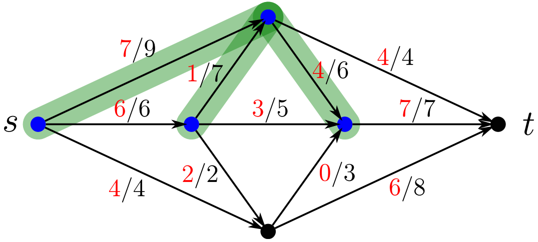
G_f



(G, s, t, c)

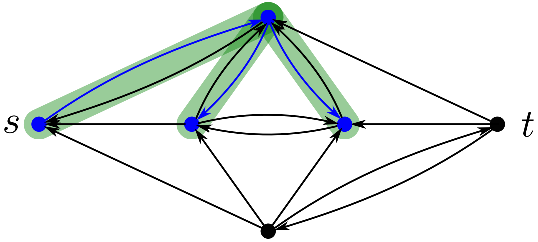
f flow

$\mathsf{v}(f) = 17$



There is no augmenting path, STOP.

G_f

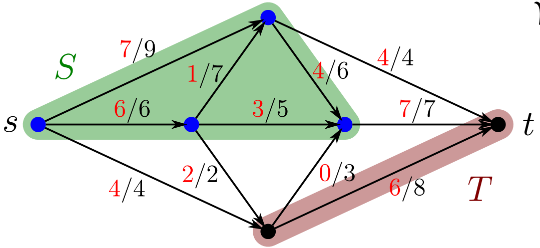


(G, s, t, c)

f flow

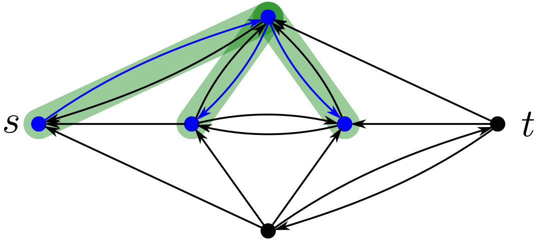
$\mathsf{v}(f) = 17$

$\mathcal{V} = \{S, T\}$



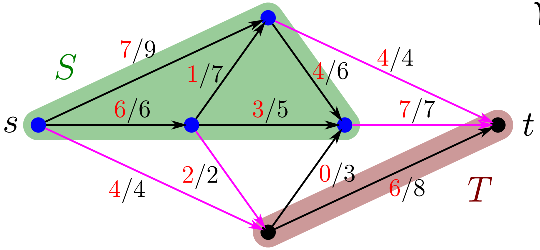
The endpoints of the augm. path candidates define a cut proving maximality.

G_f



(G, s, t, c)
f flow
$\mathsf{v}(f) = 17$

$\mathcal{V} = \{S, T\}$



The endpoints of the augm. path candidates define a cut proving maximality.

(This cut $\mathcal{V}$ is a minimum cut of the network.)

$$\max_{\mathfrak{f} \text{ flow}} \mathsf{v}(\mathfrak{f}) \leq c(\mathcal{V}) = 4 + 2 + 7 + 4 = 17 = \mathsf{v}(f).$$

$$\Downarrow$$

f is a maximum-value flow, i.e. in this network the maximum flow value is 17. $\square$