4. feladatsor – Komplex számok, Polinomok

Megoldások

1. Feladat.
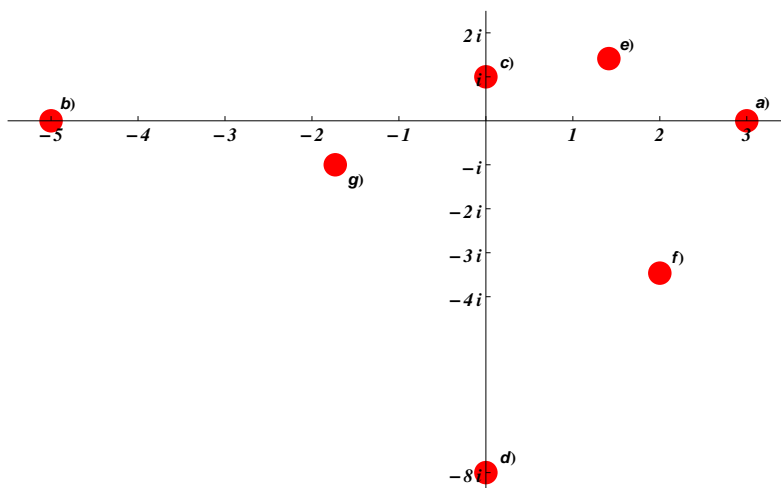
a) $-i,\ -1$ b) $41 - 11i$ c) $17 - 2i$ d) $-\frac{11}{17} + \frac{27}{17}i$ e) $-\frac{3}{13} - \frac{11}{13}i$ f) $\frac{11}{10} - \frac{23}{10}i$

2. Feladat.

a) $z = -\frac{13}{10} + \frac{11}{10}i$

b) $z = 3 - i$

c) $z_1 = \frac{3}{2},\ z_2 = 2 - i,\ z_3 = 2 + i$

d) $z_1 = \frac{8}{3},\ z_2 = 3 - i,\ z_3 = 3 + i$

e) $z_1 = 0,\ z_2 = -i,\ z_3 = \frac{\sqrt{3}}{2} + \frac{1}{2}i,\ z_4 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$
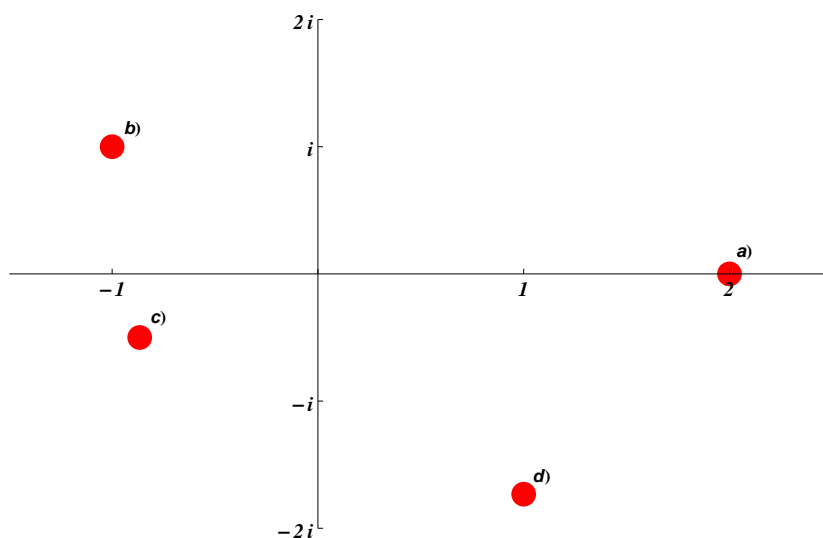
3. Feladat.

a) $3\left(\cos 0 + i \cdot \sin 0\right)$

b) $5\left(\cos \pi + i \cdot \sin \pi\right)$

c) $\cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2}$

d) $8\left(\cos\left(-\frac{\pi}{2}\right) + i \cdot \sin\left(-\frac{\pi}{2}\right)\right)$

e) $2\left(\cos \frac{\pi}{4} + i \cdot \sin\left(\frac{\pi}{4}\right)\right)$

f) $4\left(\cos\left(-\frac{\pi}{3}\right) + i \cdot \sin\left(-\frac{\pi}{3}\right)\right)$

g) $2\left(\cos\left(-\frac{5\pi}{6}\right) + i \cdot \sin\left(-\frac{5\pi}{6}\right)\right)$

4. Feladat.

a) 2

b) $-1 + i$

c) $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$

d) $1 - \sqrt{3}i$

5. Feladat.

a) $8\left(\cos\frac{\pi}{6}+i\cdot\sin\frac{\pi}{6}\right)=4\sqrt{3}+4i$

b) $\cos\left(-\frac{\pi}{2}\right)+i\cdot\sin\left(-\frac{\pi}{2}\right)=-i$

c) $\cos\left(-\frac{\pi}{6}\right)+i\cdot\sin\left(-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}-\frac{1}{2}i$

d) $2^{67}\left(\cos\frac{5\pi}{6}+i\cdot\sin\frac{5\pi}{6}\right)=-2^{66}\sqrt{3}+2^{66}i$

e) $2^{611}\left(\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}\right)=-2^{611}i$

f) $6^{1526}\left(\cos\frac{2\pi}{3}+i\cdot\sin\frac{2\pi}{3}\right)=6^{1525}\left(-3+3\sqrt{3}i\right)$

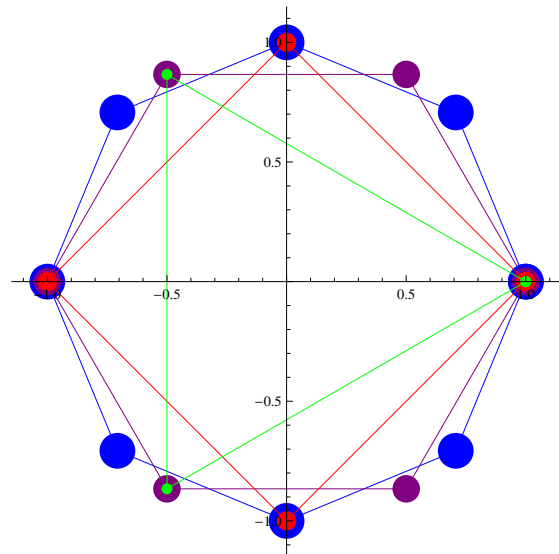## 6. Feladat.

a) $\sqrt{2}+\sqrt{2}i=2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right),$
$-\sqrt{2}+\sqrt{2}i=2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right),$
$-\sqrt{2}-\sqrt{2}i=2\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right),$
$\sqrt{2}-\sqrt{2}i=2\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right).$

b) $1+\sqrt{3}i=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right),$
$-2=2\left(\cos\pi+i\sin\pi\right),$
$1-\sqrt{3}i=2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right).$

c) $\cos\frac{\pi}{8}+i\sin\frac{\pi}{8},$
$\cos\frac{5\pi}{8}+i\sin\frac{5\pi}{8},$
$\cos\frac{9\pi}{8}+i\sin\frac{9\pi}{8},$
$\cos\frac{13\pi}{8}+i\sin\frac{13\pi}{8}.$

d) $2=2\left(\cos 0+i\sin 0\right),$
$1+\sqrt{3}i=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right),$
$-1+\sqrt{3}i=2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right),$
$-2=2\left(\cos\pi+i\sin\pi\right),$
$-1-\sqrt{3}i=2\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right),$
$1-\sqrt{3}i=2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right).$

e) $2i=2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right),$
$-\sqrt{3}-i=2\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right),$
$\sqrt{3}-i=2\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right).$

f) $\sqrt[4]{2}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right),$
$\sqrt[4]{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right),$
$\sqrt[4]{2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right),$
$\sqrt[4]{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)=\sqrt[4]{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).$

7. Feladat.

a) Harmadik egységyökök: $1,-\frac{1}{2}+\frac{\sqrt{3}}{2}i,-\frac{1}{2}-\frac{\sqrt{3}}{2}i.$
Primitív harmadik egységgyökök: $-\frac{1}{2}+\frac{\sqrt{3}}{2}i,-\frac{1}{2}-\frac{\sqrt{3}}{2}i.$

b) Negyedik egységyökök: $1,i,-1,-i.$
Primitív negyedik egységgyökök: $i,-i.$

c) Hatodik egységyökök: $1,\frac{1}{2}+\frac{\sqrt{3}}{2}i,-\frac{1}{2}+\frac{\sqrt{3}}{2}i,-1,$
$-\frac{1}{2}-\frac{\sqrt{3}}{2}i,\frac{1}{2}-\frac{\sqrt{3}}{2}i.$
Primitív hatodik egységgyökök: $\frac{1}{2}-\frac{\sqrt{3}}{2}i,\frac{1}{2}+\frac{\sqrt{3}}{2}i.$

d) Nyolcadik egységyökök: $1,\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,i,-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,-1,-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i,-i,\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i.$
Primitív nyolcadik egységgyökök: $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,$
$-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i,\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i.$

8. Feladat.

a) $x_1=-3-i,\ x_2=-3+i,\ x^2+6x+10=(x+3+i)(x+3-i)$

b) $x_1=1+2i,\ x_2=3-4i,\ x^2-(4-2i)x+11+2i=(x-1-2i)(x-3+4i)$

c) $x_1=2i,\ x_2=-\sqrt{3}-i,\ x_3=\sqrt{3}-i,\ 2x^3+16i=2(x-2i)(x+\sqrt{3}+i)(x-\sqrt{3}+i)$

d) $x_1=1+i,\ x_2=1-i,\ x_3=-1+i,\ x_4=-1-i,\ x^4+4=(x-1-i)(x-1+i)(x+1-i)(x+1+i)$

e) $x_1=\sqrt[4]{2}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right),\ x_2=\sqrt[4]{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right),\ x_3=\sqrt[4]{2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right),\ x_4=\sqrt[4]{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right),$
$x^4+1+\sqrt{3}i=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

9. Feladat.

a) $f=g\cdot\left(x^3+2x^2+3x+1\right)+4$

b) $f=g\cdot\left(x^3+3x^2-2\right)+x$

c) $f=g\cdot\left(x^3+\frac{1}{2}x+2\right)-2x-1$

10. Feladat.

a) $\text{lnko}\{f,g\}=x+1$

b) $\text{lnko}\{f,g\}=x^2+6x-7$

c) $\text{lnko}\{f,g\}=x^4-x^3+x^2-2x+1$