

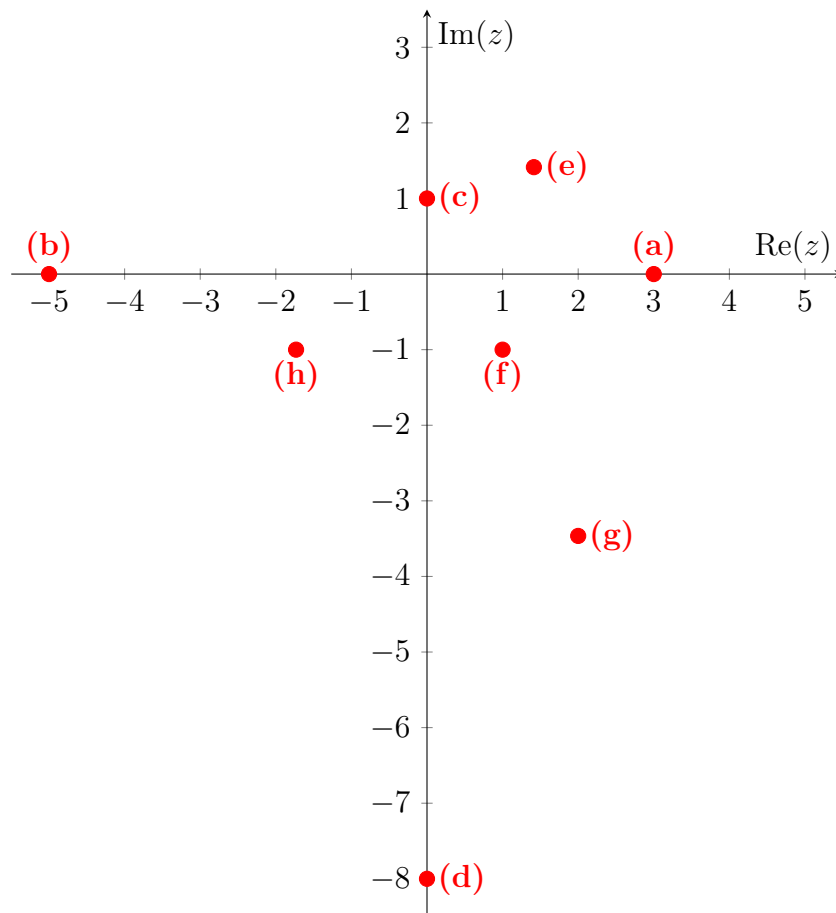
6. feladatsor – Komplex számok, polinomok

6.1. Feladat megoldása.

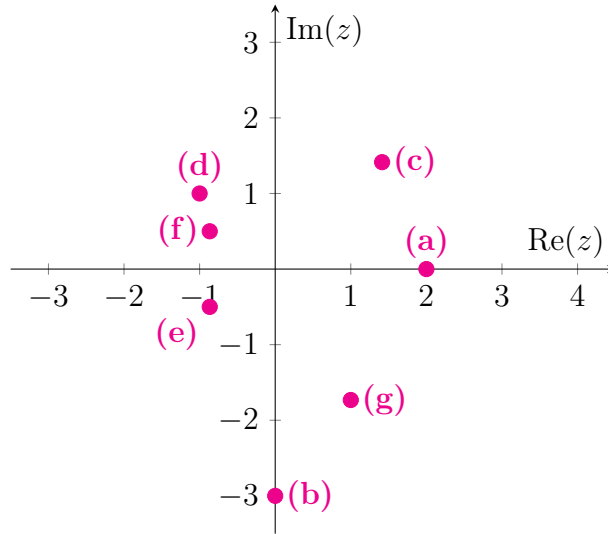
(a) $-i$; -1 ; (b) $41 - 11i$; (c) $17 - 2i$; (d) $-\frac{11}{17} + \frac{27}{17}i$; (e) $-\frac{3}{13} - \frac{11}{13}i$; (f) $\frac{11}{10} - \frac{23}{10}i$;
 (g) $-\frac{2}{5} + \frac{3}{10}i$.

6.2. Feladat megoldása.

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| <p>(a) $3 = 3 \cdot (\cos 0 + i \cdot \sin 0) = 3e^0$;</p> <p>(b) $-5 = 5 \cdot (\cos \pi + i \cdot \sin \pi) = 5e^{\pi i}$;</p> <p>(c) $i = \cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2} = e^{\frac{\pi}{2}i}$;</p> <p>(d) $-8i = 8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = 8e^{\frac{3\pi}{2}i}$;</p> | <p>(e) $\sqrt{2} + \sqrt{2}i = 2(\cos \frac{\pi}{4} + i \cdot \sin \frac{\pi}{4}) = 2e^{\frac{\pi}{4}i}$;</p> <p>(f) $1 - i = \sqrt{2}(\cos \frac{7\pi}{4} + i \cdot \sin \frac{7\pi}{4}) = \sqrt{2}e^{\frac{7\pi}{4}i}$;</p> <p>(g) $2 - 2\sqrt{3}i = 4(\cos \frac{5\pi}{3} + i \cdot \sin \frac{5\pi}{3}) = 4e^{\frac{5\pi}{3}i}$;</p> <p>(h) $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \cdot \sin \frac{7\pi}{6}) = 2e^{\frac{7\pi}{6}i}$.</p> |
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| <p>6.3. Feladat megoldása. (a) $2(\cos 0 + i \sin 0) = 2$;</p> <p>(b) $3e^{\frac{3\pi}{2}i} = -3i$;</p> <p>(c) $2e^{\frac{\pi}{4}i} = \sqrt{2} + \sqrt{2}i$;</p> | <p>(d) $\sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = -1 + i$;</p> <p>(e) $\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$;</p> <p>(f) $e^{\frac{5\pi}{6}i} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$;</p> <p>(g) $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}) = 1 - \sqrt{3}i$.</p> |
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6.4. Feladat megoldása.

- (a) $8(\cos \frac{\pi}{6} + i \cdot \sin \frac{\pi}{6}) = 4\sqrt{3} + 4i;$
 (b) $\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2} = -i;$
 (c) $\cos(-\frac{\pi}{6}) + i \cdot \sin(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2}i;$
 (d) $\cos \pi + i \cdot \sin \pi = -1;$
 (e) $2^{67}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6});$
 (f) $2^{611}(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2});$
 (g) $6^{1526}(\cos \frac{2\pi}{3} + i \cdot \sin \frac{2\pi}{3}).$

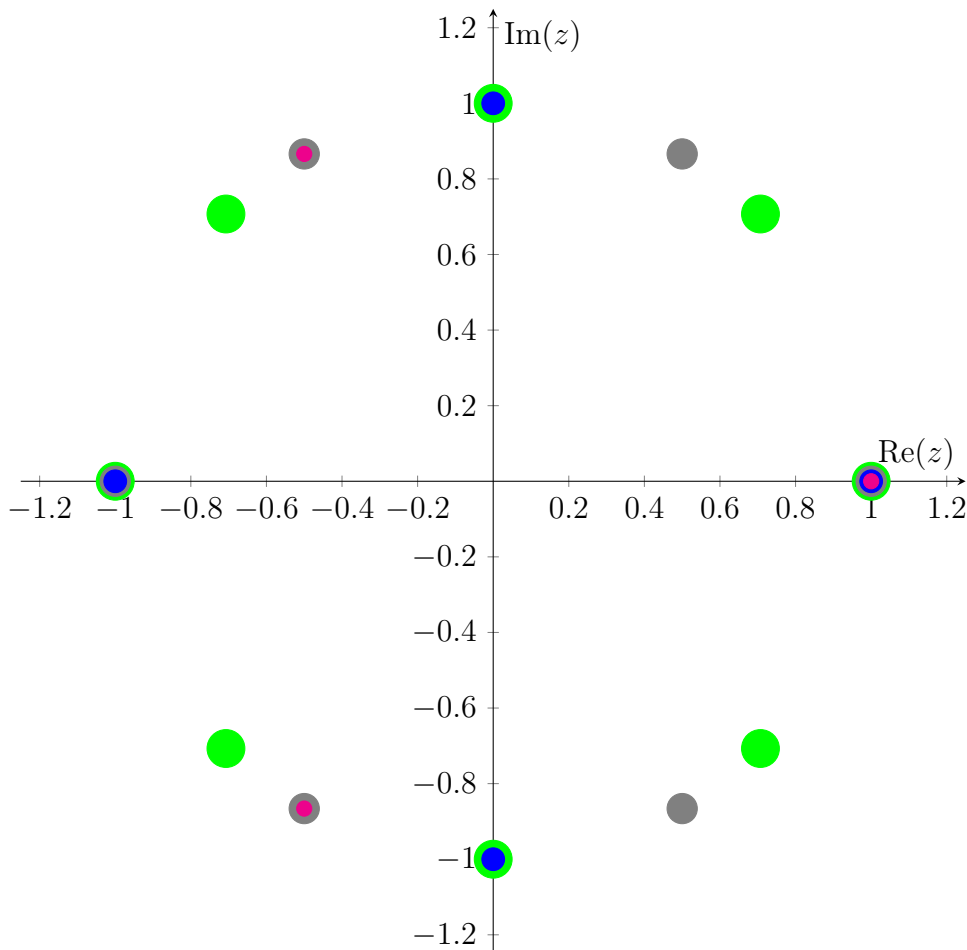
6.5. Feladat megoldása.

- (a) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$
 $-2i = 2(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$
 (b) $-2 = 2(\cos \pi + i \sin \pi)$
 $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$
 $1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3}))$
 (c) $\cos \frac{\pi}{8} + i \sin \frac{\pi}{8},$
 $\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8},$
 $\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8},$
 $\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}.$
 (d) $2 = 2(\cos 0 + i \sin 0),$
 $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}),$
 $-1 + \sqrt{3}i = 2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}),$
 $-2 = 2(\cos \pi + i \sin \pi),$
 $-1 - \sqrt{3}i = 2(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}),$
 $1 - \sqrt{3}i = 2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}).$
 (e) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}),$
 $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}),$
 $\sqrt{3} - i = 2(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}).$
 (f) $\sqrt[4]{2}(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = \sqrt[4]{2}(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}),$
 $\sqrt[4]{2}(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \sqrt[4]{2}(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}),$
 $\sqrt[4]{2}(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) = \sqrt[4]{2}(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}),$
 $\sqrt[4]{2}(\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \sqrt[4]{2}(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}).$

6.6. Feladat megoldása.

- (a) Harmadik egységgyökök: $1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$
 Primitív harmadik egységgyökök: $-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$
 (b) Negyedik egységgyökök: $1, i, -1, -i.$
 Primitív negyedik egységgyökök: $i, -i.$
 (c) Hatodik egységgyökök: $1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i.$
 Primitív hatodik egységgyökök: $\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i.$

- (d) Nyolcadik egységgyökök: $1, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -1, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.
 Primitív nyolcadik egységgyökök: $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.



6.7. Feladat megoldása.

- (a) $x_1 = -3 - i, x_2 = -3 + i$
 $x^2 + 6x + 10 = (x + 3 + i)(x + 3 - i)$
- (b) $x_1 = 2i, x_2 = -\sqrt{3} - i, x_3 = \sqrt{3} - i$
 $2x^3 + 16i = (x - 2i)(x + \sqrt{3} + i)(x - \sqrt{3} + i)$
- (c) $x_1 = -2, x_2 = 1 + \sqrt{3}i, x_3 = 1 - \sqrt{3}i$
 $x^3 + 8 = (x + 2)(x - 1 - \sqrt{3}i)(x - 1 + \sqrt{3}i)$
- (d) $x_1 = \sqrt[4]{2} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right),$
 $x_2 = \sqrt[4]{2} \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right),$
 $x_3 = \sqrt[4]{2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right),$
 $x_4 = \sqrt[4]{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = \sqrt[4]{2} \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right).$
 $x^4 + 1 + \sqrt{3}i = (x - x_1)(x - x_2)(x - x_3)(x - x_4)$

6.8. Feladat megoldása. (a) -2 ; (b) 0 ; (c) 2 ; (d) $x + 1$.

6.9. Feladat megoldása. (a) x^2 ; (b) $-x^2 + 8x - 4$; (c) $-\frac{13}{15}x^3 + 3x^2 + \frac{28}{15}x$.